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The Scalar Field Cosmology and Schrödinger Equation Formulation

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Non-linear schrödinger-formulation of cosmology is expressed here to be a useful method in cosmology. We applied the method to power-law expansion, $a \sim t^q$ with $q = 2$ in standard cosmology for a universe in which canonical phantom or non-phantom scalar field and barotropic fluid under arbitrary potential are presented. In the setup with power-law expansion, we obtain scalar field potential as function of time. The potential agrees well with result from standard cosmology method when scalar field is dominant. The method could provide an alternative procedure of solving this type of systems.

คำหลัก: Schrödinger-formulation of cosmology, Scalar field cosmology

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Abstract

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Keywords: Schrödinger-formulation of cosmology, Scalar field cosmology

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where C is integration constant. The inverse function of $\psi(x)$ exists if $P(x) \neq 0$ and $n \neq 0$. It is important for $\psi^{-1}(x)$ to exist as function since the existence of the relation $x = \sigma(t)$ (Eq. (3.5)) needs a condition,

$$x = \psi^{-1} \circ \phi(t) = \sigma(t). \tag{3.9}$$

In case that $P(x) = 0$ and $n \neq 0$, the function $\psi = C$, then inverse of ψ can exist but not as a function, i.e. one value of x corresponds to more than one value of ψ^{-1}. Indeed here one value of x corresponds to infinite value of ψ^{-1}, hence the relation (3.9) does not valid.

If the inverse function, ψ^{-1} exists (i.e. $P(x) \neq 0$ and $n \neq 0$), then the scalar field potential as function of time, $V \circ \sigma^{-1}(x)$ can be expressed as

$$V(t) = \frac{12}{\kappa^2 n^2}\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)^2 - \frac{2u^2}{\kappa^2 n}P(x) + \frac{12u^2}{\kappa^2 n^2}E + \frac{3ku^{4/n}}{\kappa^2}. \tag{3.10}$$

3.2 Remarks on the method

Although the potential obtained is not expressed as a function of ϕ, however if one can find $\dot{\phi}^2$ in Eq. (2.10) and can integrate to obtain $\phi(t)$, the solution can be substituted to the known function $V(\phi)$ motivated from fundamental physics in order to get $V(t)$. Then one can compare it to the $V(t)$ obtained from non-linear Schrödinger method. The advantage of the non-linear Schrödinger method is that it does not require us the knowledge of $V(\phi)$ which represents fundamental physics. Instead, it requires the knowledge of $a(t)$, D and k which can be directly obtained and constrained by observation. The method could be an alternative way to constrain fundamental physics.

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บทที่ 1

Introduction

1.1 Scalar field in cosmology

Scalar field plays important role in explaining inflationary phase in the early universe. The universe is observed to be in accelerating expansion at present [1, 3, 2] and scalar field is considered by scientific community to cause of observed present acceleration [4]. Alternative mathematical approaches to standard cosmology have been attempted recently. One of these is non-linear Ermakov-Pinney equation for a canonical scalar field cosmology in presence of barotropic perfect fluid [5].

There has also been a propose recently that cosmological equations for a universe with mixture of scalar field and barotropic fluid can be expressed with non-Ermakov-Milne-Pinney equation. The equations, instead, are expressed in form of non-linear Schrödinger-like equation. To obtain a successful link between these two types of equations, one needs to impose relation between functions in the Schrödinger form to major variables in cosmology. The propose and proof of the link was performed in Ref. [6]. This fact suggests that the methods could help solving problems in scalar field cosmology in alternative to standard procedure and to procedure in Ermakov-Pinney form. The method might open new way of tackling calculation in cosmology especially those to deal with scalar field in other types of Friedmann background such as braneworlds or loop quantum cosmology.

Here we investigate the method applied to the situation of power-law expansion when the scalar field and barotropic fluid are in presence and show the link between variables in the non-linear Schrödinger form and the standard cosmology. We obtain the scalar field potential as function of time. The method to obtain the scalar field potential depends only on the scalar factor, density and spatial curvature which can be from observational data.

Therefore it is advantageous that it can bring observational cosmological parameters to predict a form of scalar field potential. This could give an alternative way to constrain fundamental physics. In Sec. 2, the cosmological system is introduced. Next, in Sec. 3, we discuss how Non-linear Schrödinger formulation quantities are related to quantities in standard scalar field cosmology. We consider power-law expansion in Sec. 4 in which we show relation between Schrödinger formulation and cosmology in this case before deriving scalar field potential, Schrödinger potential and wave function. Then we give conclusions and comments to the method.

1.2 Objectives

- To obtain scalar field potential as function of time for a power-law expansion
- To investigate an alternative way to constrain fundamental physics via Schrödinger formulation
- To investigate possibility to apply the Schrödinger method to cosmology when scalar field and barotropic fluid are presented.

1.3 Scope of research

- Cosmology of scalar field and barotropic fluid
- Schrödinger formulation of standard cosmology
- Friedmann-Robertson-Walker universe
- Open, closed and flat universes

1.4 Expectation

- Obtaining scalar field potential as function of time
- Knowing advantage and disadvantage of the Schrödinger formulation in comparison to the standard cosmological method
- Agreement of the results to standard cosmological method

บทที่ 2

Cosmological equations

Considering a Friedmann-Lemaître-Robertson-Walker universe. The Einstein field equation are

$$H^2 = \frac{\kappa^2 \rho_t}{3} - \frac{k}{a^2}, \tag{2.1}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_t + 3p_t), \tag{2.2}$$

where $\kappa^2 \equiv 8\pi G = 1/M_{\rm P}^2$, k is spatial curvature, ρ_t and p_t are total density and total pressure i.e., $\rho_t = \rho_\gamma + \rho_\phi$ and $p_t = p_\gamma + p_\phi$. The barotropic component is denoted by γ, while for scalar field, by ϕ. Equations of state for barotropic fluid and scalar field are $p_\gamma = w_\gamma \rho_\gamma$ and $p_\phi = w_\phi \rho_\phi$. We consider minimally couple scalar field with Lagrangian density

$$\mathcal{L} = \frac{1}{2}\epsilon\dot{\phi}^2 + V(\phi), \tag{2.3}$$

where $\epsilon = 1$ for non-phantom case and -1 for phantom case. Density and pressure of the field are given as

$$\rho_\phi = \frac{1}{2}\epsilon\dot{\phi}^2 + V(\phi), \tag{2.4}$$

$$p_\phi = \frac{1}{2}\epsilon\dot{\phi}^2 - V(\phi), \tag{2.5}$$

therefore

$$w_\phi = \frac{\epsilon\dot{\phi}^2 - 2V(\phi)}{\epsilon\dot{\phi}^2 + 2V(\phi)}. \tag{2.6}$$

The field obeys conservation equation

$$\epsilon\left[\ddot{\phi} + 3H\dot{\phi}\right] + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0. \tag{2.7}$$

For the barotropic fluid, we set $w_\gamma \equiv (n-3)/3$ so that $n = 3(1 + w_\gamma)$, then for cosmological constant $n = 0$, for fluid at acceleration bound ($w_\gamma = -1/3$) $n = 2$,.for dust $n = 3$, for radiation $n = 4$, and for stiff fluid $n = 6$. Solution of conservation equation for the barotropic fluid is

$$\rho_\gamma = \frac{D}{a^{3(1+w_\gamma)}} = \frac{D}{a^n}, \tag{2.8}$$

then

$$p_\gamma = w_\gamma \frac{D}{a^n} = \frac{(n-3)}{3}\frac{D}{a^n}, \tag{2.9}$$

where a proportional constant $D \geq 0$. Using Eqs. (2.1), (2.4), (2.5), (2.7) and (2.8), it is straightforward to show that

$$\epsilon\dot{\phi}(t)^2 = -\frac{2}{\kappa^2}\left[\dot{H} - \frac{k}{a^2}\right] - \frac{nD}{3a^n}, \tag{2.10}$$

$$V(\phi) = \frac{3}{\kappa^2}\left[H^2 + \frac{\dot{H}}{3} + \frac{2k}{3a^2}\right] + \left(\frac{n-6}{6}\right)\frac{D}{a^n}. \tag{2.11}$$

บทที่ 3

Non-linear Schrödinger form

3.1 Correspondence between Schrödinger form and cosmology

Following the proof in [6], the corresponding non-linear Schrödinger-like equation for a standard scalar field cosmology with barotropic fluid is

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}u(x) + [E - P(x)]\,u(x) = -\frac{nk}{2}u(x)^{(4-n)/n}\,. \tag{3.1}$$

The wave function $u(x)$, the total energy E and the Schrödinger potential $P(x)$, all are on the left-hand side, are related to the standard cosmology quantities on the right-hand side as

$$\begin{aligned} u(x) &\equiv a(t)^{-n/2}\,, && (3.2)\\ E &\equiv -\frac{\kappa^2 n^2}{12}D\,, && (3.3)\\ P(x) &\equiv \frac{\kappa^2 n}{4}a(t)^n \epsilon\dot{\phi}(t)^2\,. && (3.4) \end{aligned}$$

The mapping from t to x is via

$$x = \sigma(t), \tag{3.5}$$

such that

$$\begin{aligned} \dot{\sigma}(t) &= u(x)\,, && (3.6)\\ \phi(t) &= \psi(x)\,. && (3.7) \end{aligned}$$

The function $\psi(x)$ is related to the Schrödinger potential $P(x)$ via

$$\psi(x) = \frac{2}{\kappa\sqrt{n}}\int\sqrt{P(x)}\,\mathrm{d}x + C\,, \tag{3.8}$$

บทที่ 4

Power-law expansion

Here in this section, we apply the method above to the power-law expansion in scalar field cosmology when with or without presence of barotropic fluid. The power-law expansion of the universe during inflation era,

$$a(t) = t^q \,, \tag{4.1}$$

with $q > 1$ was proposed by Lucchin and Matarrese [7] to give exponential potential

$$V(\phi) = \left[\frac{q(3q-1)}{\kappa^2 t_0^2}\right] \exp\left\{-\kappa\sqrt{\frac{2}{q}}\left[\phi(t) - \phi(t_0)\right]\right\} \,, \tag{4.2}$$

in the situation that the scalar field is the dominant component and the barotropic radiation fluid is negligible. The slow-roll parameters of the model are $\varepsilon = 1/q$ and $\eta = 2/q$. The power spectrum index is

$$n_{\rm PS} = 1 - 6\varepsilon + 2\eta \,. \tag{4.3}$$

For scale invariant spectrum $n_{\rm PS} \simeq 0$ hence $q = 2$ for scale invariant spectrum [8].

4.1 Relating Schrödinger quantities to scalar field cosmology

The wave function in Schrödinger form is therefore related to cosmology as

$$u(x) = \dot{\sigma}(t) = t^{-qn/2} \,. \tag{4.4}$$

We can integrate the equation above so that the Schrödinger scale x is related to the time scale as

$$x = \sigma(t) = -\frac{t^{-\beta}}{\beta} + \tau \,, \tag{4.5}$$

where $\beta \equiv (qn-2)/2 > 0$ and τ is an integrating constant The parameter x and t have the same dimension since β is a number. Using Eq. (4.1), we can find $\epsilon\dot{\phi}(t)^2$ from Eq. (2.10):

$$\epsilon\dot{\phi}(t)^2 = \frac{2q}{\kappa^2 t^2} + \frac{2k}{\kappa^2 t^{2q}} - \frac{nD}{3t^{qn}} . \tag{4.6}$$

We use Eqs. (4.1) and (4.6) in Eq. (3.4), therefore the Schrödinger potential is found to be

$$P(x) = \frac{qn}{2} t^{qn-2} + \frac{kn}{2} t^{q(n-2)} - \frac{\kappa^2 n^2 D}{12} . \tag{4.7}$$

4.2 Scalar field potential $V(t)$

In order to obtain $V(t)$ in Eq. (4.9), we need to know derivative of $u(x)$:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x} u(x) &= \frac{\mathrm{d}}{\mathrm{d}\left[-(t^{-\beta}/\beta) + \tau\right]} t^{-qn/2} , \\ &= -\beta \frac{\mathrm{d}}{\mathrm{d}R} R^{1+1/\beta} , \\ &= -\frac{qn}{2t} , \end{aligned} \tag{4.8}$$

where we set a variable $R \equiv t^{-\beta}$ for helping in integration. At this step, using Eqs. (3.2), (3.3), (3.4) and (4.8) in Eq. (4.9), we finally obtain

$$V(t) = \frac{q(3q-1)}{\kappa^2 t^2} + \frac{2k}{\kappa^2 t^{2q}} + \left(\frac{n-6}{6}\right) \frac{D}{t^{qn}} . \tag{4.9}$$

In Figs. 4.1 and 4.2, we assume flat universe ($k = 0$) and $q = 2$. When there is no barotropic fluid, the potential $V(t)$ obtained from the new method (the red line) matches the one solved from standard cosmology (the points). The green line is the case when the dust is presented with scalar field. The same for the blue line but radiation instead of dust. The result is regardless of the value of ϵ.

In phantom case $\epsilon = -1$, the solutions ϕ are imaginary. In the non-phantom case, although $\epsilon = 1$, when $D \neq 0$, it is not always possible to integrate to obtain real-valued solution $\phi(t)$. The numerical integration results shown in Fig. 4.2 are of the case $D = 0, k = 0$ which is $\phi(t) = (\sqrt{2q}/\kappa)\ln(t)$ and of the case $D \neq 0, k = 0$ with dust component ($n = 3$). The solution $\phi(t) = (\sqrt{2q}/\kappa)\ln(t)$ of the case $D = 0, k = 0$ is used in Eq. (4.9) so that $V(\phi)$ is found to be the same as Eq. (4.2) when $t_0 = 1$ and $\phi(t_0) = 0$ confirming the result found in [7].

Fig. 4.3 presents a comparative illustration of $V(t)$ obtained from non-linear Schrödinger method for closed, flat and open universe when the fluid components are dust and scalar field. If without dust, i.e. the scalar field is the only dominant component, the results look similar to the case $D = 0$ in Fig. 4.1. When the components are radiation and scalar field, it is not always possible to get the result since $\phi(t)$ could be imaginary as mentioned before.

4.3 Schrödinger potential $P(x)$

We can get Schrödinger potential $P(x)$ from Eqs. (4.5) and (4.7) where time is expressed as a function of x as

$$t(x) = \frac{1}{[-\beta(x-\tau)]^{1/\beta}}. \tag{4.10}$$

Therefore

$$\begin{aligned} P(x) &= \frac{2qn}{(qn-2)^2}\frac{1}{(x-\tau)^2} \\ &+ \frac{kn}{2}\left[\frac{-2}{(qn-2)(x-\tau)}\right]^{2q(n-2)/(qn-2)} \\ &- \frac{\kappa^2 n^2 D}{12}. \end{aligned} \tag{4.11}$$

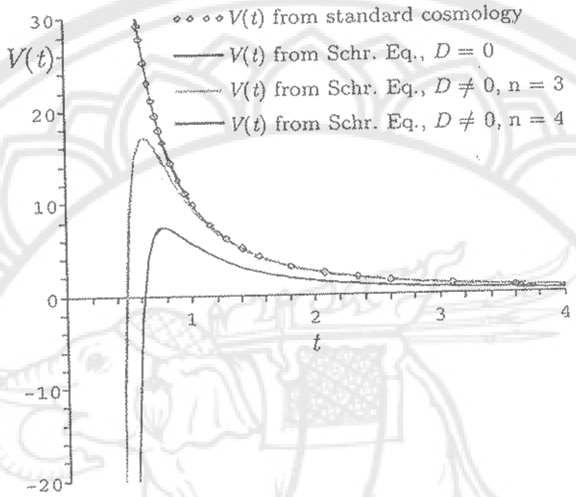
A disadvantage of Eq. (4.11) is that when we can not use it in the case of scalar field domination as applied to inflationary expansion. Dropping D term in Eq. (4.11) by setting $D = 0$ can not be considered as scalar field domination case since coefficient n of the barotropic fluid equation of state still appears in the other terms. The Schrödinger potentials $P(x)$ plotted with x for power-law expansion with $q = 2$ in closed, flat and open universe are shown in Fig. 4.4. In the figure, the dust cases are shown on the right and radiation cases are on the left. We set $\kappa = 1, D = 1.0$ and $\tau = 0$.

4.4 Schrödinger wave function $u(x)$

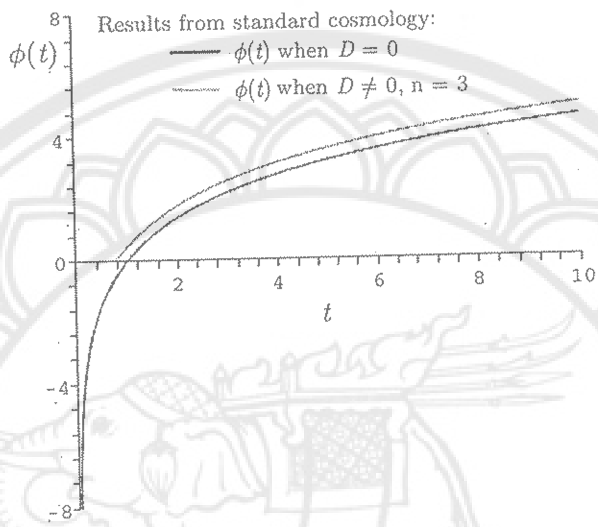
The wave function can be directly found from Eqs. (4.4) and (4.10) as

$$u(x) = \left[\left(-\frac{1}{2}qn+1\right)(x-\tau)\right]^{qn/(qn-2)}, \tag{4.12}$$

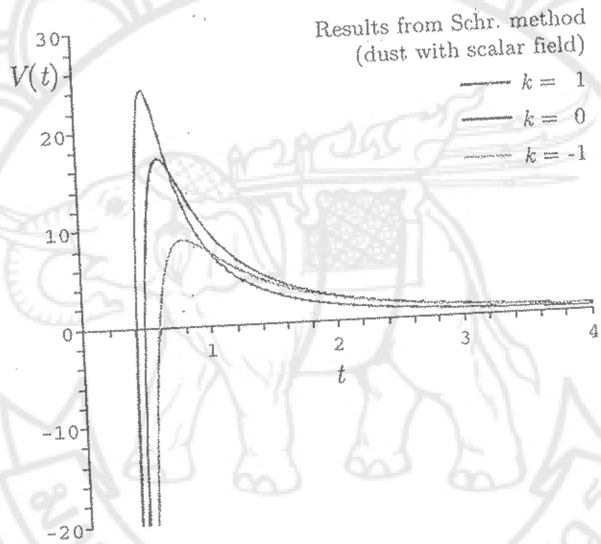
which is independent of the spatial curvature k or the initial density D. However, coefficient n of the barotropic fluid equation of state and q must be expressed. The Schrödinger method is therefore efficient in case that there are both scalar field and a barotropic fluid in presence together. Wave functions for a range of barotropic fluid are presented in Fig. 4.5. The result is confirmed by substituting Eq. (4.12) into Eq. (3.1).



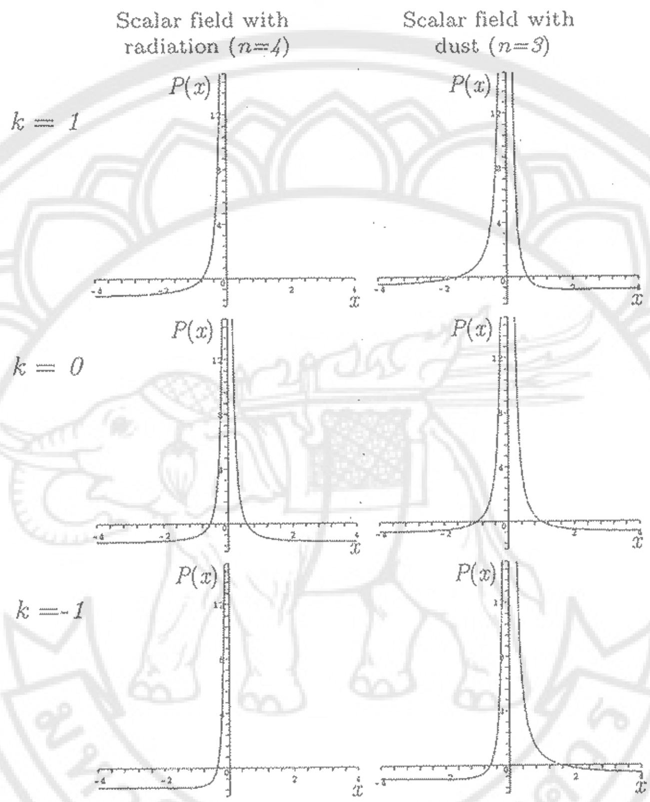
รูปที่ 4.1: Potential $V(t)$ plots from the standard cosmology method and non-linear Schrödinger method for power-law expansion $a \sim t^q$, $q = 2$ in flat universe ($k = 0$). The point-plot is of the potential (4.2) which is solved directly in standard cosmology in Ref. [7] when the barotropic fluid density is negligible. The red line is of $V(t)$ obtained from the non-linear Schrödinger method when the barotropic fluid density is set to zero $D = 0$. The green line is of $V(t)$ obtained from the non-linear Schrödinger method when there is also dust fluid together with scalar field, i.e. $D \neq 0$ and $n = 3$. The blue line obtained from the non-linear Schrödinger method when the universe has scalar field with radiation fluid, i.e. $D \neq 0$ and $n = 4$. Here we set $\kappa = 1$ and in the last two plots, $D = 1$.



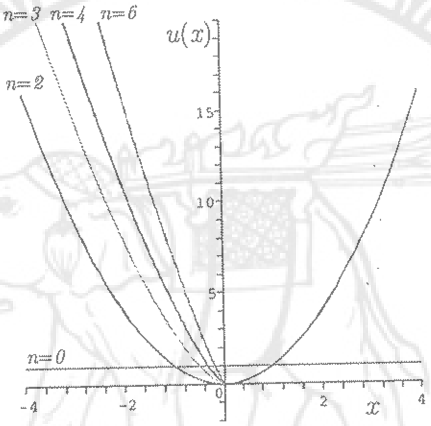
รูปที่ 4.2: $\phi(t)$ plots from the standard cosmology method for power-law expansion $a \sim t^q$, $q = 2$ in flat universe ($k = 0$). The red line is of the when the barotropic fluid density is negligible. The green line is in the presence of scalar field with dust ($D \neq 0$ and $n = 3$). In the figure, $\kappa = 1$ and $D = 1$.



รูปที่ 4.3: $V(t)$ obtained from non-linear Schrödinger method for closed, flat and open universe in presence of dust and scalar field.



รูปที่ 4.4: $P(x)$ plotted versus x for power-law expansion. Here $q = 2$. We set $\kappa = 1, D = 1.0$ and $\tau = 0$. The scalar field dominant case can not be plotted since even we set a condition $D = 0$, coefficient n of the barotropic fluid equation of state is still in the first and second terms of Eq. (4.11). There is only a real-value $P(x)$ for the cases $k = \pm 1$ with $n = 4$ because, when $x > 0$, $P(x)$ becomes imaginary in these cases.



รูปที่ 4.5: $u(x)$ plotted versus x for power-law expansion with $q = 2$. We set $\tau = 0$. The wave function is plotted for $n = 0$ (cosmological constant), $n = 2$, $n = 3$ (dust), $n = 4$ (radiation) and $n = 6$ (stiff fluid). There is no real-value wave function for $n = 3$, $n = 4$ and $n = 6$ unless $x < 0$.

บทที่ 5

Conclusions and Comments

We have applied the Schrödinger-type formulation to power-law expansion scalar field cosmology in presence of barotropic fluid. We show the link between cosmological quantities and Schrödinger quantities and then obtain scalar field potential $V(t)$, Schrödinger potential $P(x)$ and wave function $u(x)$. In the case of a scalar field dominant in flat universe, our analytical result agrees well with the result in [7]. A range of plots is presented in various cases including when the universe is closed, flat or open. In Sec. 4 we begin with the Schrödinger method by assuming how scale factor a relates to time t and evaluate other following quantities to finally obtain $V(t)$. One might wonder if we start from quantum mechanics by solving the non-linear Schrödinger equation (3.1). The equation can be simplified to linear type if we consider the flat universe case $k = 0$ or the case $n = 2$ or $n = 4$ [6]. However, in performing the calculation, $P(x)$ (Eq. (3.4)) must be known and it depends explicitly on $a(t)$ and $\dot{\phi}$ (Eq. (2.10)) which as well depends on $a(t)$ through H. Therefore this method also depends on how we assume law of expansion $a(t)$ and knowing $a(t)$ enables us to know $u(x)$ directly (see Eq. (4.4)). Then we do not need to solve the Schrödinger equation.

The method is suitable for studying a system of scalar field dark energy and dark matter since it requires a presence of both scalar field and a barotropic fluid. At late time the scalar field dark energy and cold dark matter are two major components of the universe while the others are negligible. The method needs to assume the knowledge of $a(t)$, k and D which are observable in order to find $V(t)$. Although it is better to know $V(\phi)$ so that it can directly relate to fundamental physics. However if ones start from fundamental physics with a particular potential $V(\phi)$ and if they know how ϕ evolves with t then V can be expressed as function of t. As a result, $V(t)$ results from observation and another from fundamental physics can be compared to each other. This could be an interesting further work.

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