



TRAVERSABLE WORMHOLES IN MASSIVE GRAVITY THEORY



A Thesis Submitted to the Graduate School of Naresuan University
in Partial Fulfillment of the Requirements
for the Master of Science in (Theoretical Physics)

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has been approved by the Graduate School as partial fulfillment of the requirements
for the Master of Science in Theoretical Physics of Naresuan University

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ABSTRACT

Morris-Thorne wormhole or traversable wormhole (TWH) is wormhole which we are physically allowed to travel through. The main problem of this wormhole is that the exotic matter is needed as a constituent of the wormhole. The exotic matter is unknown for us with strong negative radial pressure, known here as wormhole tension. In general relativity (GR), negative pressure plays a role of wormhole tension, forbidding travelling through such a wormhole. The same problem is also found in a wormhole solution for GR with a cosmological constant. Here we consider it in the context of a modified gravitational theory so called massive gravity. Although, the massive gravity is constructed aiming in explaining the cosmic acceleration, the wormhole solution exists in this theory. Moreover, there are many free parameters in wormhole solution for the de Rham-Gabadadze-Tolley (dRGT) massive gravity. The necessity of the exotic matter can be alleviated. As a result, the wormhole with lower size of throat, tension as well as volume of exotic matter can be obtained in this work.

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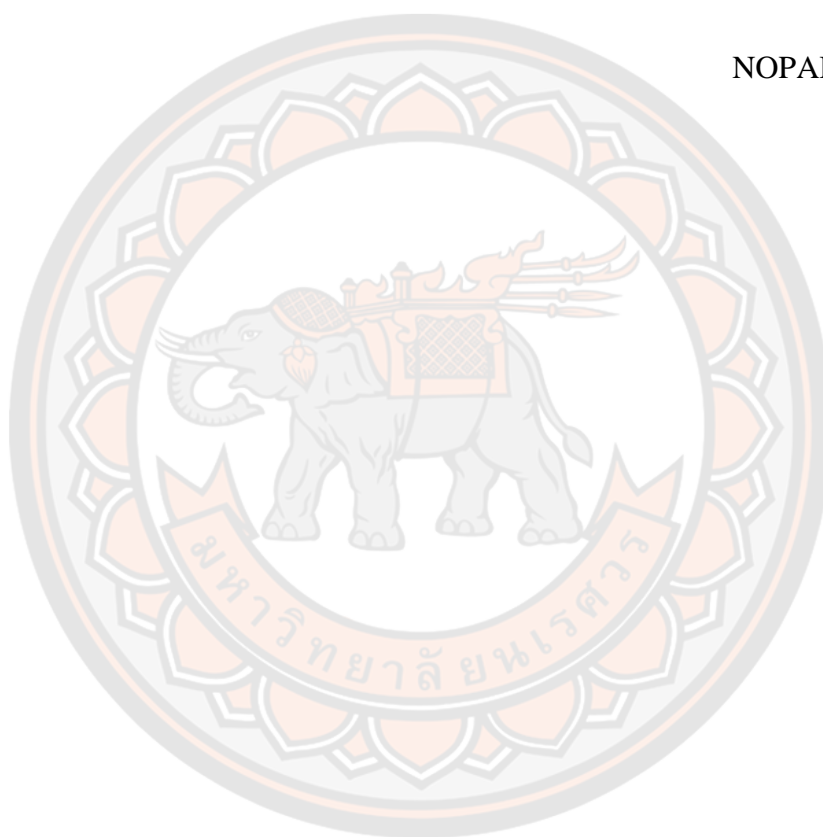


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CHAPTER I

INTRODUCTION

Background and Motivation

In general relativity, wormhole is a region which connects between two sheets of spacetime. In Schwarzschild solution (Hobson, Efstathiou, & Lasenby, 2006), a wormhole solution exists but a traveling through this wormhole violates causality. Moreover, there is a horizon as a region with infinite redshift surface so that the wormhole doesn't appear. Thus, an existence of wormholes corresponds to a condition as a redshift function must be finite. In addition, there is the other problem in which human cannot travel through the wormhole. Therefore, Mike Morris and Kip Thorne (Morris & Thorne, 1988), they found conditions to solve this problem by specifying conditions for construction of traversable wormholes, called traversability conditions as follows weak tidal force, weak gravity, and short traveling time.

However, a construction of wormholes must be used a matter to generate curved spacetime for the wormhole. Exotic property of the matter is identified by defining a parameter associated with the violation of null energy condition (NEC). For the existence of the throat of the wormhole, it must be used exotic matter in which radial pressure is negative.

In physical system, there exists phenomenon of the violations (Visser, 1995) i.e. Casimir effect, cosmological inflation, etc., but they would be difficult to be found and to construct the wormhole. Thus, the good choice is the wormhole without exotic matter. However, existence of the throat of the wormhole imposes that the matter near the throat must be exotic. For another choice, it is minimized to use of the exotic matter as well as it should be a weak violation for some class of solution. One of mitigations for a minimization of the exotic matter is using a thin shell formalism which is a formalism to construct a surface layer between regions of spacetime, proposed by Matt Visser (Visser, 1989a, 1989b). Moreover, by using the thin shell formalism, one can produce the redshift function in exterior solution to be a constant which corresponds to the existence of wormholes.

One of the interesting properties is the accelerating expansion of the universe because it corresponds to the negative pressure. Consequently, we can expect to solve the problem for the wormhole by including cosmological model.

At the same time, a theory of gravitation is modified because general relativity cannot explain the accelerating expansion of the universe. In simple modified gravity, the wormholes are investigated by Francisco S. N. Lobo (Lobo, 2007). In this model, the negative pressure can be reduced. Moreover, there is a solution of the traversable wormhole without thin shell (junction conditions of boundary surface between the matter and empty space). For some class of de Sitter solution corresponding to the accelerating expansion of the universe, it can minimize the use of exotic matter (Lemos, Lobo, & de Oliveira, 2003). As a result, the study of traversable wormholes in modified gravity theory is an important option which is possible to discover the wormholes in cosmology.

One of interesting modified gravity theories is massive gravity theory, a theory in which a graviton acquires non-zero mass. The recent viable model of the theory is massive gravity with nonlinear interaction terms, known as dRGT (de Rham-Gabadadze-Tolley) massive gravity (De Rham & Gabadadze, 2010; De Rham, Gabadadze, & Tolley, 2011). The interesting result of this theory is that the accelerating expansion can be obtained. Moreover, there are several parameters which provide a description different from the cosmological constant. Therefore, the wormhole solution in dRGT massive gravity is more general than de Sitter and Anti de Sitter solution, while the dRGT massive gravity solution can be reduced to de Sitter and Anti de Sitter solution to be some range of the model parameters.

CHAPTER II

LITERATURE REVIEW

Wormholes

In 1916, motivated by General relativity, Flamm (Flamm, 1916) proposed a geometry that some curve might connect two regions of spacetime but it was only a concept of model in physics. Later, Hermann Weyl proposed a theory about a matter in which a space is connected by mass and analysis of electromagnetic field energy in 1921 (Weyl, 1921) and explained it by philosophy of mathematics and natural science (Weyl, 2009) in 1928. It is therefore anticipated that the matter can take an object from one place to another.

In General relativity, there is a problem from Schwarzschild solution which a particle cannot escape from black hole. Thus, Einstein and Rosen built a geometrical model to solve this problem and found a bridge across a double-sheet of spacetime in 1935 (Einstein & Rosen, 1935). However, this model is just theoretical. The time-like particle cannot pass through the bridge, due to the violation of causality.

Because of topological structure with connected spacetime, “Wormhole” is named by Wheeler in 1957 (Misner & Wheeler, 1957; J. Wheeler, 1962). Wheeler envisage wormhole as the fabric of the universe, a chaotic sub-atomic realm of quantum fluctuations, which he called “Quantum foam” (J. A. Wheeler, 1955).

In 1988, Morris and Thorne defined a redshift function and a shape function to analyze a horizon and a throat of the wormhole. They proposed a wormhole solution which is possible for a human’s traveling, called “Traversable wormhole” (Morris & Thorne, 1988). However, it is unusable because a matter, used to construct the wormhole, is exotic.

Schwarzschild solution

In 1915, Albert Einstein proposed general relativity (Hobson et al., 2006) that is a gravitational theory explaining the geometry of spacetime by the Einstein field equation. The Einstein field equation is an equation to represent a relation between a curvature of spacetime and a matter. The Einstein field equation can be written as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}, \quad (1)$$

where $g_{\mu\nu}$ is a metric tensor, $R_{\mu\nu}$ is Ricci tensor, R is Ricci scalar, and κ is a constant of Newtonian limit. In the equation (1), Einstein tensor, $G_{\mu\nu}$, contains second derivatives of the metric and the energy-momentum tensor, $T_{\mu\nu}$, corresponds to the matter in the spacetime.

The Einstein field equation is a complicated nonlinear differential equation. It is not easy to analytically solve without imposing any assumptions. In 1916, assuming spherical symmetry, the Einstein field equation in vacuum was solved by Karl Schwarzschild (Hobson et al., 2006). Schwarzschild solution can be expressed as

$$ds^2 = -c^2 \left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (2)$$

Where c is a speed of light, G is Newton's constant, and M is mass. From the Schwarzschild metric in the equation (2), the event horizon is defined by $r_s = \frac{2GM}{c^2} = 2\mu$, which is also called "Schwarzschild radius". This radius at a surface in which light cannot escape. This property is known as a property of black holes (Schwarzschild radius is now the black hole horizon).

The spacetime curvature is described by a contraction of curvature tensor $R_{\mu\nu\rho\sigma}$. Let us consider a curvature scalar $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ (Hobson et al., 2006). This scalar at any point for the Schwarzschild metric can be computed as

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{48\mu^2}{r^5}. \quad (3)$$

It is seen that the above curvature scalar diverges at $r = 0$ but it is still finite at $r = 2\mu$. Consequently, the singularity at $r = 2\mu$ is actually the coordinate singularity arisen from using improper coordinates. The way to eliminate this singularity was proposed by Martin Kruskal and George Szekeres in 1960 (Carroll, 2004). They transformed the coordinates to the new ones without the coordinate singularity, $r =$

2μ . By fixing angular coordinates ($\theta = \frac{\pi}{2}$), the metric in Kruskal coordinates can be obtained as

$$ds^2 = -\frac{32\mu^3}{r} e^{-\frac{r}{2\mu}} (dV^2 - dU^2). \quad (4)$$

The spacetime diagram in Kruskal coordinates can be divided in four regions as shown in figure 1.

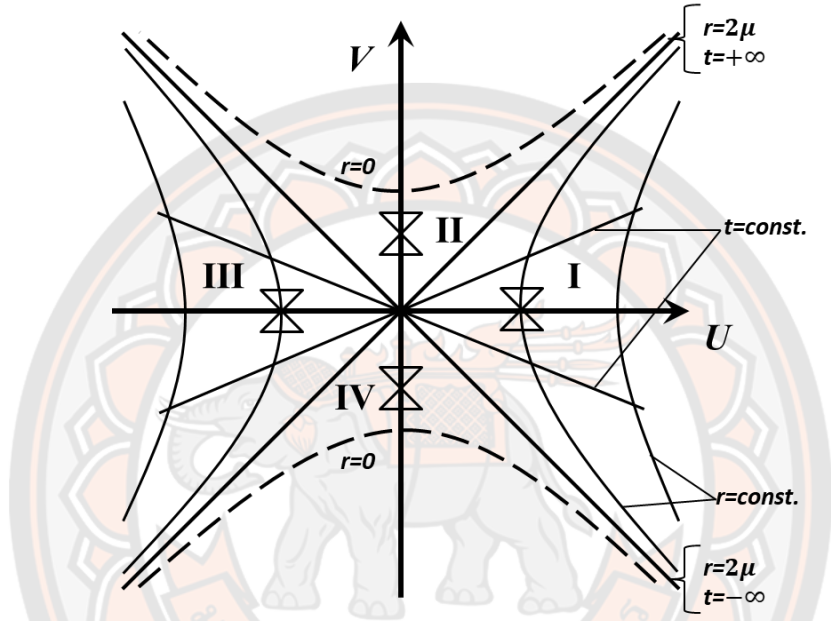


Figure 1 spacetime diagram in Kruskal coordinates.

From the diagram in figure 1, the region I and III are asymptotically flat region, the region II is black hole, and the region IV is inverse of region II known as white hole. So, it is found that there is a connected region between the regions I and III. This is motivated to solve the particle problem in General relativity (Einstein & Rosen, 1935) by Rosen and Einstein in 1935. They built a geometrical model of a physical elementary particle in which everywhere is finite and singularity free. The Schwarzschild solution was embedded to the axisymmetric coordinate system in three-dimensional Euclidean space as embedding diagram shown in figure 2. They also found that there exists a connected region between two sheets of spacetime, called “Einstein-Rosen bridge”. Unfortunately, it is impossible to travel because the bridge is pinched off before the particle passes through it.

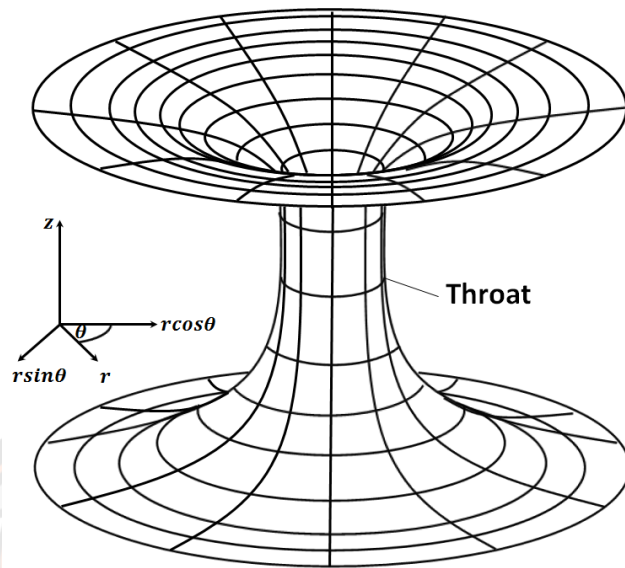


Figure 2 Einstein-Rosen bridge with embedding diagram.

Later, Roger Penrose proposed compact conformal diagram (Hawking & Ellis, 1973) which is included all regions of spacetime and supposed to preserve the light cone angle so that this is a good diagram to explain the connected region. For Schwarzschild solution, there exist the regions I and III which are asymptotically flat spacetime as shown in figure 3. In addition, regions II and IV are a black hole and a white hole, respectively.

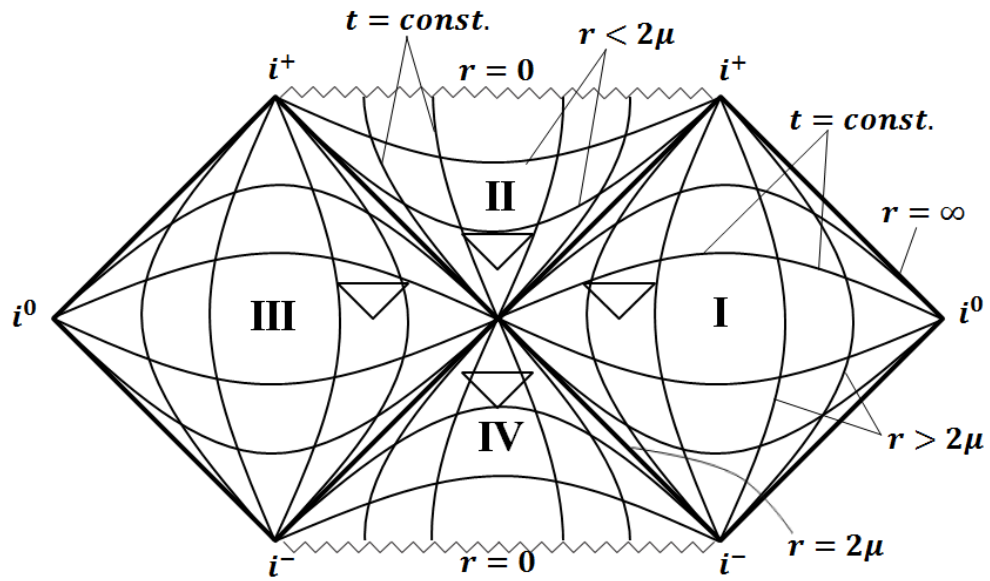


Figure 3 Conformal diagram of Schwarzschild wormhole or Einstein-Rosen bridge

For regions I and III, there exists a region which connects between two asymptotically flat spacetimes, called Schwarzschild wormhole. In the diagram, there are triangles of light cones with slope ± 1 . A trajectory of a time-like particle must be inside these triangles (the slope of trajectory must be greater than one of light cone) so that the particle cannot travel from regions I to region III and vice versa. This is called the causality problem.

Traversable wormholes

Morris and Thorne proposed a “Traversable wormhole” (Morris & Thorne, 1988) to solve a traveling problem of wormhole so that traversable wormhole is a wormhole in which human can travel through. The Schwarzschild wormhole cannot be a traversable wormhole because a traveling through the wormhole violates causality. It is notice that the Schwarzschild solution is vacuum solution. It is possible to construct the traversable wormhole if we consider the solution with a matter. Traversable wormhole is assumed to be spherically symmetric and static, similar to the Schwarzschild wormhole. Morris and Thorne investigated a suitable form of the matter in order to construct the wormhole. The traversable wormhole solution is solved by specifying conditions to eliminate the traveling problem.

General static wormhole metric

From the Schwarzschild solution, Morris and Thorne analyzed and defined new functions by considering properties of the metric for the spherical symmetric wormholes. The metric can be expressed as

$$ds^2 = -e^{2\Phi(r)} dt^2 + \left(1 - \frac{b(r)}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (5)$$

where $\Phi = \Phi(r)$ and $b = b(r)$ are two arbitrary functions, to be constrained by the following properties. From the Schwarzschild metric, the component of metric g_{00} relates to gravitational redshift (Hobson et al., 2006) as

$$\frac{\nu_R}{\nu_E} = \left(\frac{g_{00}(r_E)}{g_{00}(r_R)}\right)^{\frac{1}{2}}, \quad (6)$$

where parameters ν_R and ν_E are frequency of wave as observed by receiver and emitter respectively at their radius r_R and r_E . So, the function that $\Phi(r)$ determines the gravitational potential, called the redshift function. Moreover, the shape of throat of wormholes depends on the function $b(r)$ in the metric component g_{11} . For the Schwarzschild solution in embedding form, a relation of the coordinates z and r (Hobson et al., 2006) can be written as

$$\frac{dz}{dr} = \pm \left(\frac{r}{2\mu} - 1\right)^{-\frac{1}{2}}, \quad (7)$$

where $b(r) = 2\mu$ in the Schwarzschild case. Hence, the function $b(r)$ determines the spatial shape of the wormhole, called the shape function.

Throat of wormholes

In the wormhole solution, there must exist a throat for connecting double-sheet of spacetime. Substituting the shape function, $b(r) = 2\mu$ into the equation (7), one obtains

$$\frac{dz}{dr} = \pm \left(\frac{r}{b(r)} - 1 \right)^{-\frac{1}{2}}. \quad (8)$$

The throat (Morris & Thorne, 1988) is equivalent to a minimum radius defined by shape function that $b(r_o) = r_o = b_o$ where a constant b_o is the throat interpreted as the minimum radius r_o .

The throat of wormhole is defined as the shape function at minimum radius (Morris & Thorne, 1988). $b(r_o) = r_o = b_o$, where b_o is a constant interpreted as the throat of wormhole.

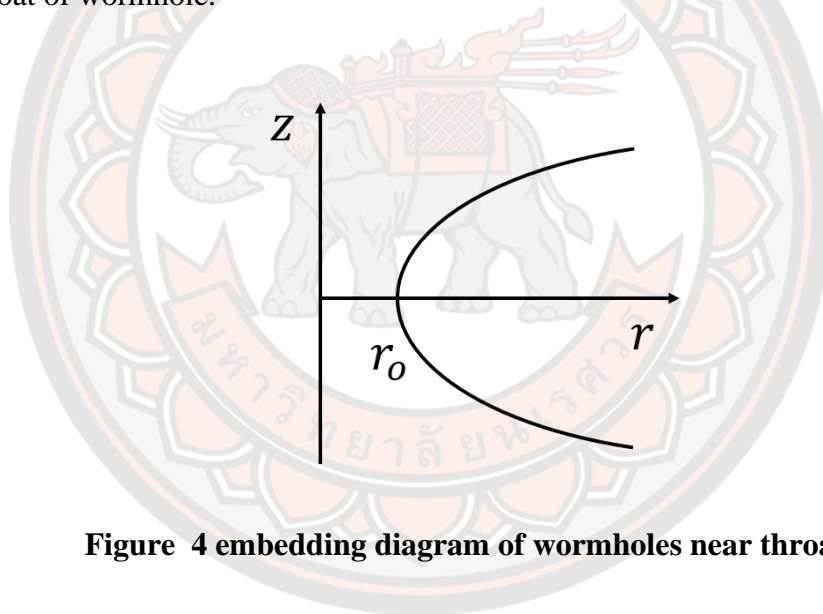


Figure 4 embedding diagram of wormholes near throat.

Integrating the equation (8), the embedding diagram near throat can be shown in figure 4. This result shows that a shape in the diagram of wormholes near throat must be approximated to only this form.

Horizon of wormholes

Although the wormhole solution exists, it cannot be observed because of existence of horizon. Anything falls into the horizon cannot be sent back. Matt Visser defined the horizon as a one-way membrane which permits the passage of light and matter in only one direction and time slows to a stop at the horizon (Visser, 1995). On the other hand, Morris and Thorne defined the horizon as the infinite redshift surface (Morris & Thorne, 1988). A traveler inside the horizon cannot send any information to outside. If the wormhole has the horizon, it does not appear. The horizon in this case can be defined from a metric component as

$$g_{00} = -e^{2\Phi} \rightarrow 0. \quad (9)$$

From equation (9), if the horizon does not appear, the function Φ must be (negative) infinite. As a result, the existence of wormholes is defined that the redshift function must be finite throughout the journey through wormholes.

An event horizon is defined as a boundary of region from which casual curve can reach asymptotic null infinity (Visser, 1995). Thus, the event horizon can be obtained by

$$g^{11}(r_{EH}) = 0,$$

where r_{EH} is a radius at the event horizon. However, this horizon is the same as event horizon in only static solution since a time-like particle can escape from this horizon in rotating solution.

Exotic parameter

The most important issue of traversable wormholes is finding suitable form of a matter in order to construct the wormholes. In general relativity, the spacetime is curved when there exists mass so that the matter must be used to construction a wormhole.

The energy momentum tensor of the matter satisfying spherical symmetry (Morris & Thorne, 1988) is defined by

$$T_{\mu\nu} = \text{diag}(\rho, -\tau, p, p). \quad (10)$$

where τ is tension, negative radial pressure. This definition corresponds to a perfect fluid which is a simple matter in nature.

One of the important energy conditions is the null energy condition (NEC) (Visser, 1995). The definition of NEC for any null vector is

$$T_{\mu\nu}k^\mu k^\nu > 0. \quad (11)$$

In terms of the principal pressures,

$$\forall_i, \rho + p_i \geq 0. \quad (12)$$

The exotic matter violates of NEC. Therefore, a parameter to characterize the violation is defined as (Morris & Thorne, 1988)

$$\xi = \frac{\tau - \rho c^2}{|\rho c^2|}, \quad (13)$$

which is dimensionless and must be less than zero for ordinary matter. The important part of wormholes is the throat so that we consider the parameter from definition of throat. Considering the (0,0)- and (1,1)-components of the Einstein field equation, the parameter in equation (13) can be rewritten as follows

$$\xi = \frac{2b^2}{r|\rho c^2|} \left(\frac{d^2 r}{dz^2} \right) - 2r \left(1 - \frac{b}{r} \right) \frac{\phi'}{|\rho c^2|}. \quad (14)$$

From the existence of the throat, one obtains $\left(\frac{d^2 r}{dz^2} \right) > 0$. In addition, the last term converges to zero because $r = b$ at the throat so that one obtains

$$\xi > 0. \quad (15)$$

From equation (15), the matter near throat must be exotic. In physical system, there are violations of energy condition so that it is acceptable by acquiring low negative pressure close to zero.

Traversability conditions

The traversability conditions [6 (Lobo, 2007)] are based on physical limit of human to travel through the wormhole as follows. The first condition is a weak tidal force. The gravitational force in traversable wormhole must be the same order of magnitude for the gravitational force on the earth. The components of Riemann tensors, corresponding to longitude and transverse tidal forces, can be written respectively as

$$R_{010}^1 = e^\Phi \left(1 - \frac{b}{r}\right) \left(\Phi'' - \frac{1}{2} \frac{(rb' - b)}{r^2 \left(1 - \frac{b}{r}\right)} \Phi' + (\Phi')^2\right) \lesssim g_\oplus / (c^2 h_{\text{human}}), \quad (16)$$

and

$$R_{020}^2 = \frac{\gamma^2}{2r^2} \left(\left(\frac{v}{c}\right)^2 \left(b' - \frac{b}{r}\right) + 2(r - b)\Phi'\right) \lesssim g_\oplus / (c^2 h_{\text{human}}). \quad (17)$$

The second condition corresponds to the first condition which is weak gravity condition,

$$|\Phi' c^2| \lesssim g_\oplus. \quad (18)$$

The third condition is short travelling time condition. The traversable wormholes are modified from the Schwarzschild wormhole by promoting a constant $b = 2\mu$ to be $b = b(r)$. The constant in the Schwarzschild wormhole is a value of minimum radius at the wormhole throat. As a result, the throat is pinched off before any particle can cross. For the wormhole metric, the shape function $b(r)$ corresponds to the properties of the throat as well as the travelling time of the traveler.

For the wormholes in general case, the radial coordinate is ill-behaved near the throat. Thus, a proper radial distance measured by the traveler, is obtained by

$$l(r) = \pm \int_{r_{\min}}^r \frac{dr}{\left(1 - \frac{b(r)}{r}\right)^{\frac{1}{2}}}. \quad (19)$$

The proper radial distance relates to travelling time of the traveler. Hence, the travelling time is required to be less than order of one year as

$$\Delta\tau = \int_{-l_1}^{l_2} \frac{dl}{v\gamma} \lesssim 1yr, \quad \Delta t = \int_{-l_1}^{l_2} \frac{dl}{ve^\Phi} \lesssim 1yr, \quad (20)$$

where τ is time of traveler, t is time of observer at rest, v is a velocity of a spaceship, and γ is Lorentz factor. These conditions would be used to investigate the possibility to travel through the wormholes.

Thin shell formalism

In 1989, Matt Visser proposed an improvement of wormhole structure by using “Thin shell formalism” (Poisson & Visser, 1995; Visser, 1995). The thin shell formalism is a formalism which constructs a connected membrane between regions. This is a technique of general relativity for analyzing gravitational field of the matter in a layer. Therefore, the thin shell determines a shell between the matter and empty spacetime. From the Einstein field equation, the shell can be considered as a delta-function. An infinitesimal integral of the field equation through spherical thin shell can be written as (Lobo, 2007)

$$\int_{-}^{+} G_{\mu\nu} dr = \kappa \int_{-}^{+} t_{\mu\nu} \delta(r - a) dr, \quad (21)$$

where $t_{\mu\nu}$ is the energy momentum tensor of thin shell around a surface radius a . Note that the integral in equation (21) is preformed from inside surface $-$ to outside surface $+$. Then, using a property of δ -function, one obtains

$$t_{\mu\nu} = \frac{1}{\kappa} \int_{-}^{+} G_{\mu\nu} dr. \quad (22)$$

The components of energy momentum tensor contain surface energy Σ and a surface tangential pressure \mathcal{P} . For this formalism, a radius of the thin shell surface can be specified. In addition, it is possible to minimize the use of exotic matter.

Morris-Thorne wormholes with a cosmological constant

In 2010, traversable wormholes for General relativity with a cosmological constant are studied by José P. S. Lemos, Francisco S. N. Lobo, and Sérgio Quinet de Oliveira (Lobo, 2007). Including the cosmological constant term to Einstein field equation, the equation can be rewritten as

$$G_{\mu\nu} - g_{\mu\nu}\Lambda = \kappa T_{\mu\nu} \quad (23)$$

where Λ is the cosmological constant. General relativity with the cosmological constant is a modified gravity theory which can provide the accelerating expansion of the universe. Therefore, it is interesting to analyze the wormholes in this theory.

The exotic matter is considered by using the parameter defined in the equation (13). However, by analyzing the field equation, the cosmological constant does not affect the parameter. So, the matter near a throat in this theory is still exotic.

The wormhole solution can be divided into three parts as follows; interior solution, exterior solution, and junction between interior and exterior parts. The interior solution satisfies a constant redshift function. As a result, by using (0,0)-component of the field equation, the tension at the throat can be written as

$$\tau(r_o) = \frac{1}{\kappa} \left(\frac{1}{r_o^2} + \Lambda \right). \quad (24)$$

For de Sitter case $\Lambda < 0$, the tension can be positive, zero, and negative but for another case, $\Lambda \geq 0$, the tension is only positive. Therefore, for de Sitter case, the exotic matter can be minimized. However, the tension is still positive by considering the parameter at the throat.

The exterior solution is a solution in empty spacetime so that this solution corresponds to black hole solution. Solving the field equation in empty spacetime, one obtains a vacuum solution with a cosmological constant so that the event horizon can be obtained by solving an equation

$$g^{11} = \left(1 - \frac{2\mu}{r} + \frac{\Lambda}{3}r^2 \right) = 0. \quad (25)$$

By solving this equation, there are two event horizons for de Sitter case and one event horizon for the case of $\Lambda \geq 0$. For de Sitter case, the interior event horizon is event horizon of black hole and exterior event horizon is cosmological event horizon. So if we live between two horizons, we can see the accelerating expansion at large scale.

For appearance of wormholes, a surface radius a of the shell must be larger than the inner horizon of wormhole.

The junction is a thin shell for matching the metric between interior solution and exterior solution as well as it can minimize the use of exotic matter. For this wormhole, one obtains an energy density of thin shell as

$$\Sigma = 0 \quad (26)$$

and a tangential pressure of thin shell as

$$\mathcal{P} = \frac{1}{\kappa} \frac{\left(\frac{\mu}{a} + \frac{\Lambda}{3}a^2\right)}{\sqrt{1 - \frac{2\mu}{a} + \frac{\Lambda}{3}a^2}}. \quad (27)$$

The pressure in de Sitter case can be positive, zero, and negative depending on the surface and the cosmological constant, shown in figure 5, but in another case $\Lambda \geq 0$, the pressure is only positive.

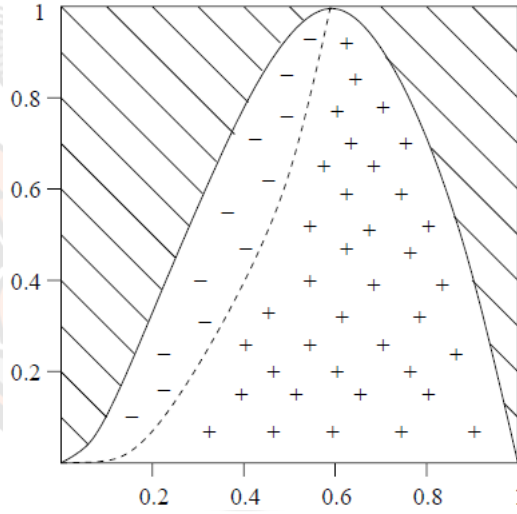


Figure 5 Tangential pressure of thin shell in de Sitter case plot between $-9\Lambda\mu^2$ and $2\mu/a$.

Then, matching (0,0)-component of the field equation between interior and exterior parts, one can obtain a radial pressure in terms of the tangential pressure as

$$\tau(a) = \frac{2}{a} \mathcal{P} e^{\Phi(a)}. \quad (28)$$

Assuming zero pressure, we found that a surface radius is larger than the horizon. As a result, the solution in de Sitter case corresponds to the wormhole without thin shell.

Moreover, from the equation (24), the use of exotic matter can be minimized.

Massive gravity

One of interesting modified gravity theories is a massive gravity theory in which, graviton acquires non-zero mass. The massive gravity theory started from study of linearized general relativity with interaction terms. In general relativity, the graviton is considered to be a massless particle. Fierz and Pauli (Fierz & Pauli, 1939) tried to propose a theoretical model in 1939 that the graviton is a massive particle. They introduced interaction terms to linearized General relativity. Then, it was found by van Dam, Veltman, and Zakharov that approaching the limit of the graviton mass to zero, the theory is not continuously reduced to general relativity, called “vDVZ discontinuity” (Veltman & Dam, 1970; Zakharov, 1970).

In 1972, Vainshtein showed that the linearized massive gravity cannot be trusted within some radius, known as “Vainshtein radius” (Vainshtein, 1972). He suggested that nonlinear terms must be added to the theory. At the same time, Boulware and Deser found that large class of nonlinear massive gravity theory contains a ghost instability, called BD (Boulware-Deser) ghost (Boulware & Deser, 1972).

In 2010, de Rham, Gabadadze, and Tolley introduced nonlinear interaction form which is free from the BD ghost, known as “dRGT massive gravity” (De Rham et al., 2011). The action of dRGT massive gravity can be written as

$$S = \int d^4x \sqrt{-g} \frac{1}{2\kappa^2} \left(R + m_g^2 U(g, \phi^\alpha) \right), \quad (29)$$

where R is Ricci scalar and U is a potential, m_g interpreted as graviton mass and κ is constant related to Newtonian gravitational constant. The potential in four dimensions is defined by

$$U = U_2 + \alpha_3 U_3 + \alpha_4 U_4 \quad (30)$$

where

$$U_2 = [K]^2 - [K^2],$$

$$U_3 = [K]^3 - 3[K][K^2] + 2[K^3],$$

$$U_4 = [K]^4 - 6[K]^2[K^2] + 8[K][K^3] + 3[K^2]^2 - 6[K^4].$$

α_3 and α_4 are free parameters. The square bracket denotes the traces of the matrix

$[K] = K^\mu_\mu$ and $[K^n] = (K^n)^\mu_\mu$, n is an integer, where $[K^2] = K^\mu_\nu K^\nu_\mu$, $[K^3] = K^\mu_\nu K^\nu_\rho K^\rho_\mu$, and $[K^4] = K^\mu_\nu K^\nu_\rho K^\rho_\sigma K^\sigma_\mu$. The matrix $K_{\mu\nu}$ is defined as

$$K_\nu^\mu = \delta_\nu^\mu - \sqrt{g^{\mu\sigma} f_{ab} \partial_\sigma \phi^a \partial_\nu \phi^b} \quad (31)$$

where f_{ab} is a fiducial metric and ϕ^a is Stuckelberg scalar field.

Varying the action in equation (29) with respect to the metric tensor $g_{\mu\nu}$, the field equation can be obtained as (Ghosh, Tannukij, & Wongjun, 2016)

$$G_{\mu\nu} + m_g^2 X_{\mu\nu} = 0, \quad (32)$$

where $G_{\mu\nu}$ is Einstein tensor and $X_{\mu\nu}$ is effective energy-momentum tensor. The

Einstein tensor and the effective energy-momentum tensor can be written as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R,$$

where $R_{\mu\nu}$ is Ricci tensor and

$$X_{\mu\nu} = K_{\mu\nu} - K g_{\mu\nu} - \alpha \left(K_{\mu\nu}^2 - K K_{\mu\nu} - \frac{1}{2} U_2 g_{\mu\nu} \right) + 3\beta \left(K_{\mu\nu}^3 - K K_{\mu\nu}^2 + \frac{1}{2} K_{\mu\nu} U_2 - \frac{1}{6} U_3 g_{\mu\nu} \right).$$

The parameters are redefined as $\alpha_3 = \frac{\alpha-1}{3}$ and $\alpha_4 = \frac{\beta}{4} + \frac{1-\alpha}{12}$.

The fiducial metric which can be chosen in order to find the exact solution is given by

$$f_{\mu\nu} = \text{diag}(0, 0, c^2, c^2 \sin^2 \theta) \quad (33)$$

where C is a constant. This fiducial metric is used to find a static black hole solution (Ghosh *et al.*, 2016) so that it appropriates to find a static wormhole solution by using this form of the fiducial metric.

CHAPTER III

TRAVERSABLE WORMHOLES IN DRGT MASSIVE GRAVITY

The wormhole structure consists of three main regions which are the exterior, interior regions and thin shell. The exterior region is just the empty spacetime around the wormhole. The interior region is the region where the matter field exists. These exterior and interior regions can be separated by a surface boundary (filled with some matter) called the thin shell. This shell can be thought as a container which contains the matter inside the wormhole. In addition, there is another important part of the interior region which is called the throat of the wormhole. The existence of the throat is one of the conditions for traversability.

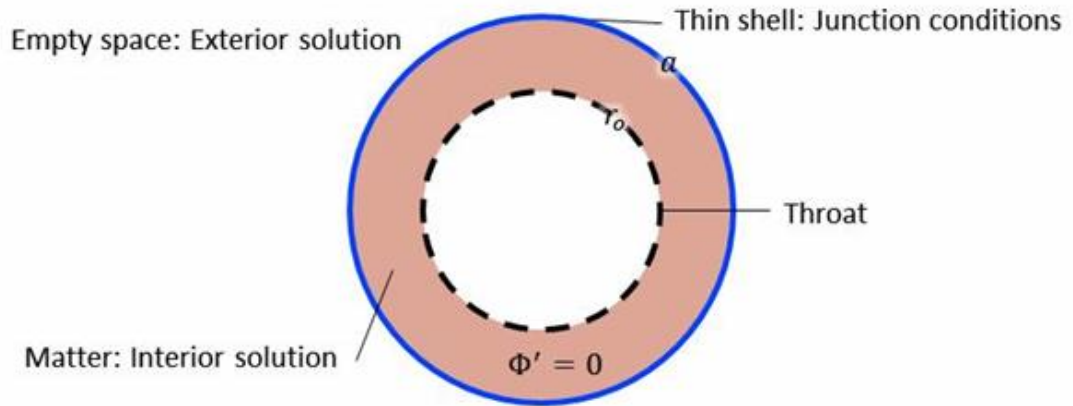


Figure 6 Structure of traversable wormhole

In this section, we will find the wormhole solution in the dRGT massive gravity theory, i.e., the exterior, interior solutions and the junction conditions for matching both solutions. Let us start by considering the static and spherically symmetric metric in the form of suitable functions can be expressed as follows

$$ds^2 = -e^{2\Phi(r)} dt^2 + \left(1 - \frac{b(r)}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad (34)$$

where Φ is a redshift function related to a gravitational potential and b is a shape function described shape of wormholes.

The exterior solution

For the exterior solution, we solve the field equation in empty spacetime $T_{\mu\nu} = 0$. The (0,0)-, (1,1)- and (2,2)-components of the field equation can be expressed, respectively, as

$$\begin{aligned} 0 &= \frac{b'}{r^2} + \Lambda + \frac{2\gamma}{r} + \frac{\zeta}{r^2}, \\ 0 &= \frac{b}{r^3} - 2\left(1 - \frac{b}{r}\right)\frac{\Phi'}{r} + \Lambda + \frac{2\gamma}{r} + \frac{\zeta}{r^2}, \\ 0 &= \left(1 - \frac{b}{r}\right)\left(\Phi'' + (\Phi')^2 - \frac{b'r - b}{2r^2\left(1 - \frac{b}{r}\right)}\Phi' - \frac{b'r - b}{2r^3\left(1 - \frac{b}{r}\right)} + \frac{\Phi'}{r}\right) + \Lambda + \frac{2\gamma}{r} + \frac{\zeta}{r^2}, \end{aligned}$$

where Λ , γ , and ζ are free parameters which can be written as α and β . $\Lambda < 0$ and $\Lambda > 0$ correspond to de Sitter and anti-de Sitter respectively. The prime denotes derivative respect to r . Moreover, it is noted that the (3,3)-component of the field equation can be reduced to the (2,2)-component. Integrating the equation, one obtains

$$\begin{aligned} b'(r) &= -(\Lambda r^2 + 2\gamma r + \zeta), \\ b(r) &= -\left(\frac{\Lambda}{3}r^3 + \gamma r^2 + \zeta r\right) + \text{Const.} \end{aligned}$$

Then, we take the limit in which the function $b(r)$ reduces to the Schwarzschild solution. So, the constant of integration reads

$$b(r) = 2\mu - \left(\frac{\Lambda}{3}r^2 + \gamma r + \zeta\right)r, \quad (35)$$

where μ is the Schwarzschild mass function. As seen in the Schwarzschild solution, g_{00} and g_{11} are inverse to each other. There exists another relation between two components of metric g_{00} and g_{11} ,

$$\begin{aligned} g_{00} &= \frac{1}{g_{11}} \\ e^{2\Phi} &= \left(1 - \frac{b}{r}\right) \\ \Phi &= \frac{1}{2}\ln\left(1 - \frac{b}{r}\right) \end{aligned}$$

Substituting b into the equation, one obtains

$$\Phi(r) = \frac{1}{2}\ln\left(1 - \frac{2\mu}{r} - \left(\frac{\Lambda}{3}r^2 + \gamma r + \zeta\right)\right). \quad (36)$$

The functions $b(r)$ and $\Phi(r)$ are very important because they can characterize the structure of the wormhole. The shape function describes how many event horizons exists while the derivative of the red shift function describes the strength of gravitational force.

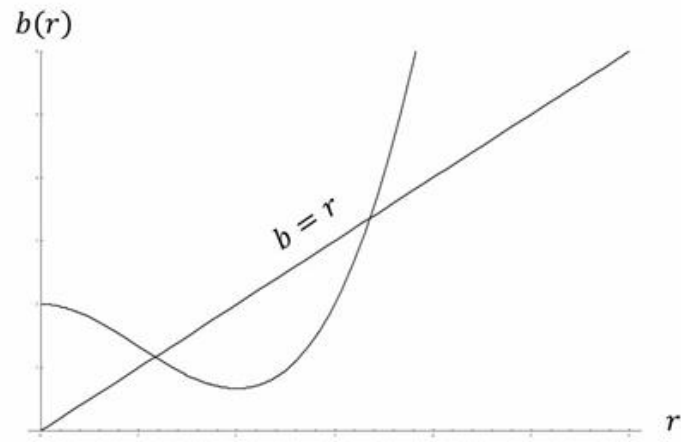


Figure 7 Shape function diagram in de Sitter-like massive gravity ($\Lambda < 0$).

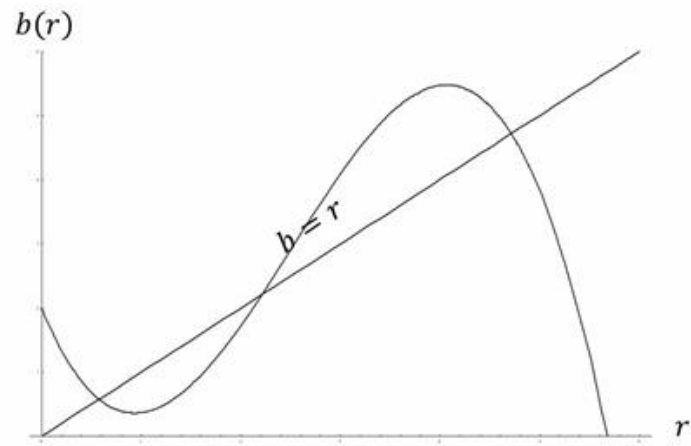


Figure 8 Shape function diagram in Anti-de Sitter-like massive gravity ($\Lambda > 0$).

The interior solution

In this case, we consider the energy momentum tensor of exotic matter which takes almost the same form as one for the perfect fluid except that radial pressure becomes negative. This negative pressure is also called tension. This energy-momentum tensor can be expressed as

$$T_{\mu\nu} = \text{diag}(\rho, -\tau, p, p) \quad (37)$$

where ρ , p , and τ are the energy or mass, pressure, and tension (negative radial pressure) respectively. For the existence of the matter, the field equation becomes

$$G_{\mu\nu} + m_g^2 X_{\mu\nu} = \kappa T_{\mu\nu}. \quad (38)$$

Let us recall one of the conditions for traversable wormhole which is that the redshift function $\Phi(r)$ must be finite. For convenience and simple, let Φ be a constant ($\Phi' = 0$). This means the zero gravity of matter which implies that the wormhole has weak gravity and weak tidal force. By this setting, the (0,0)-, (1,1)- and (2,2)-components of the field equation become (set $\kappa = 1$), respectively, as

$$\rho(r) = \frac{b'}{r^2} + \Lambda + \frac{2\gamma}{r} + \frac{\zeta}{r^2}, \quad (39)$$

$$\tau(r) = \frac{b}{r^3} + \Lambda + \frac{2\gamma}{r} + \frac{\zeta}{r^2}, \quad (40)$$

$$p(r) = \left(1 - \frac{b}{r}\right) \left(\frac{b'r - b}{2r^3(1 - \frac{b}{r})} \right) - \left(\Lambda + \frac{2\gamma}{r} + \frac{\zeta}{r^2} \right). \quad (41)$$

At the throat of the wormhole ($r = r_0$), the shape function must be satisfied $b(r_0) = r_0$. The tension at the throat in the equation becomes

$$\tau(r_0) = \frac{1}{r_0^2} + \Lambda + \frac{2\gamma}{r_0} + \frac{\zeta}{r_0^2} \quad (42)$$

It is seen that there are 3 parameters. In the next section, we will find the values of each parameters which give the appropriate tension and property of throat of wormhole.

The junction conditions

We now want to match the interior and exterior solutions at the thin shell with suitable junction conditions. There are two conditions for matching the solutions which are to match the metric and to match the equations. The later matching condition requires the existence of the surface boundary (thin shell) which separates matter field region and empty spacetime.

For the first matching condition, the metric $g_{\mu\nu}$ for the interior solution and the exterior solution are specified at the surface boundary to be continuous. From the structure of the wormhole, the matter distributes from $r = r_0$ to $r = a$, where a is a radius of mouth of the wormhole. The mouth is connected to the thin shell. Therefore, the metric matching conditions at the mouth a can be obtained as $g_{tt(ext)} = g_{tt(int)}$ and $g_{rr(ext)} = g_{rr(int)}$. This leads to match functions $\Phi_{ext}(a) = \Phi_{int}(a)$ and $b_{ext}(a) = b_{int}(a)$. As a result, the solutions of the redshift and shape functions at the thin shell can be expressed, respectively, as

$$\Phi(a) = \frac{1}{2} \ln \left(1 - \frac{2\mu}{a} - \left(\frac{\Lambda}{3} a^2 + \gamma a + \zeta \right) \right), \quad (43)$$

$$b(a) = 2\mu - \left(\frac{\Lambda}{3} a^2 + \gamma a + \zeta \right) a. \quad (44)$$

For the second condition, the equation matching refers to connect the field equations between the interior and the exterior solutions. However, a fringe of the matter remain exists. So, the thin shell formalism must be used in order to eliminate the fringe. In this formalism, the energy momentum tensor of the thin shell is

$$t_{\mu\nu} = \int_{-}^{+} G_{\mu\nu} d\hat{r} \quad (45)$$

where \hat{r} is a unit vector of radial direction in the thin shell. The limits of integration $-$ and $+$ are the value of \hat{r} at the middle between interior region-thin shell and thin shell-exterior region, respectively. The functions $\Phi(r)$ and $b(r)$ must be continuous and smooth in the thin shell. The solutions obtained from integration of Φ' and b' then vanish. As a result, the right hand side of (0,0)-component of the equation is zero. So, $t_{00} = 0$, which means that the thin shell has no mass or energy. Similarly, t_{11} also equals zero. However, in (2,2)-component of the equation, there is a term Φ'' that t_{22} does not vanish. This component can be solved as

$$t_{22} = P = \sqrt{1 - \frac{b(a)}{a}} \Phi'(a)|_{-}^{+}, \quad (46)$$

where P is a tangential or surface pressure. As mentioned previously, one has $\Phi'^{-} = 0$ for interior solution. In the exterior solution, differentiating the first expression of the equation,

$$\Phi'^{+} = \left(1 - \frac{b(a)}{a} \right)^{-1} \left(\frac{\mu}{a^2} + \frac{\Lambda}{3} a + \frac{\gamma}{2} \right). \quad (47)$$

Eventually, the tangential pressure of TWH in dRGT model is

$$\mathcal{P} = \frac{\left(\frac{\mu}{a} + \frac{\Lambda}{3} a^2 + \frac{\gamma}{2} a \right)}{\sqrt{1 - \frac{2\mu}{a} + \frac{\Lambda}{3} a^2 + \gamma a + \zeta}}. \quad (48)$$

From the above expression, one can plot for the values of surface pressure with various values of γ and ζ . In the following plots, the negative and positive values and are shown as the grey and violet regions respectively. The line separated the grey and violet regions is the zero values. The white region is the region for absence of wormhole. The vertical and horizontal axes of the plots are the radius of mouth of wormhole, a , and Λ respectively.

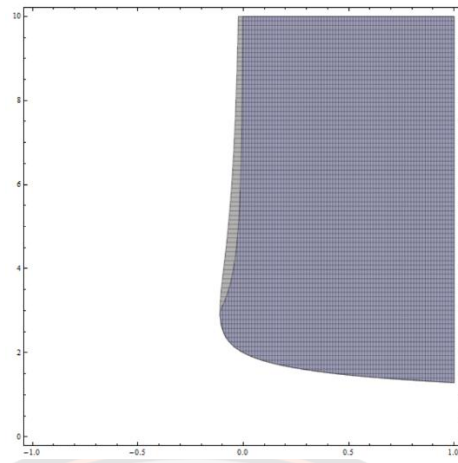


Figure 9 Surface pressure diagram with $\gamma = 0$ and $\zeta = 0$.

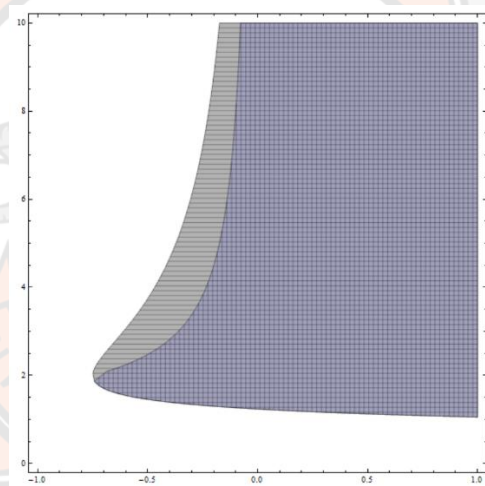


Figure 10 Surface pressure diagram with $\gamma = 0.5$ and $\zeta = 0$.

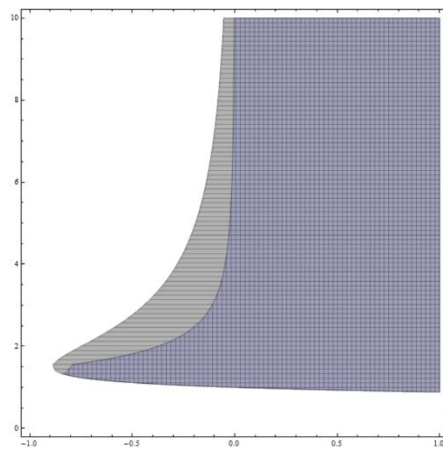


Figure 11 Surface pressure diagram with $\gamma = 0$ and $\zeta = 1$.

It is seen that there are many parameters in this solution. Hence, it is possible to find the values of parameters which correspond to the larger regions for existence of the wormhole as well as smaller wormhole. One can see the difference between wormhole solutions in dRGT massive gravity and one in GR with cosmological constant (the case for $\gamma = 0$ and $\zeta = 0$).

The signs of tangential pressure depend on the types of matter inside the thin shell. From the above expression, one can see that there exists a finite value of a for $P = 0$. This means that it is not necessary to use the thin shell of other matter for TWH in dRGT model. It is one of the advantages of this model compared to GR.



CHAPTER IV

RESULT AND DISCUSSION

Even though, TWH in dRGT massive gravity can be constructed for both asymptotically dS ($\Lambda < 0$) and AdS ($\Lambda > 0$) spacetimes. In this work, we are interested only in the asymptotically dS case because it is consistent with observations in cosmology. Moreover, the asymptotically dS case is more convenient to compare the results with the case of GR with a cosmological constant because TWH in that case exists only for dS spacetime. The tensions near the throat of TWH in dRGT massive gravity described by the equation (42) can be plotted comparing to pure GR and GR with a cosmological constant as figure.

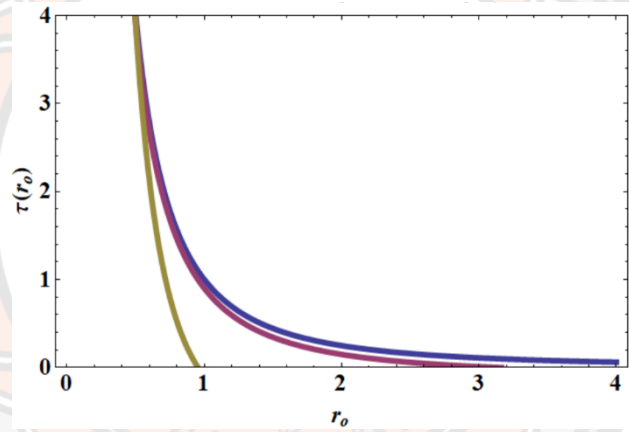


Figure 12 Plots of tensions at the throat versus the throat distance for TWH in GR (blue), GR with cosmological constant (red) ($\Lambda = -0.1$) and dRGT model (yellow) ($\Lambda = -0.1, \gamma = -1, \zeta = 1$).

The size of the throat should be small because if it is very large, it should be observed at the present. From the figure, we can see that TWH with small tension for GR case is very large ($r_o \rightarrow \infty$ as $\tau(r_o) \rightarrow 0$). However, the tensions at the throat approach to zero for finite r_o in both GR with a cosmological constant and dRGT cases.

As discussed in previously, the existence of the throat requires the function $b(r)$ satisfied the condition $b(r_o) = r_o$. The form of this function is not unique. The chosen form denoted as $b_{int}(r)$ must be consistent with the equations $\rho - p$. By matching the interior and exterior shape functions such as $b_{int}(a) = b_{ext}(a)$, the throat r_o can be found in term of a . The mouth, a , is computed from the equation of tangential pressure with the simple condition: the tangential pressure is zero, $P = 0$

that means form of the matter of TWH without the thin shell. Therefore, a volume of the exotic matter in wormhole can be computed by $V_{min} = \frac{4}{3}\pi(a^3 - r_o^3)$. We compute the volume of exotic matter for two different forms of shape functions as follows

First shape function

$$b(r) = (r_o r)^{1/2}$$

Let $P = 0$ (maximum surface radius), surface radius of wormholes with a cosmological constant can be obtained

$$a_c = \left(-\frac{3\mu}{\Lambda}\right)^{1/3}$$

The throat of wormholes with a cosmological constant can be obtained

$$r_{oc} = (-3\mu)^{5/3} \Lambda^{1/3}$$

For existence of wormholes ($a_c \geq r_{oc}$)

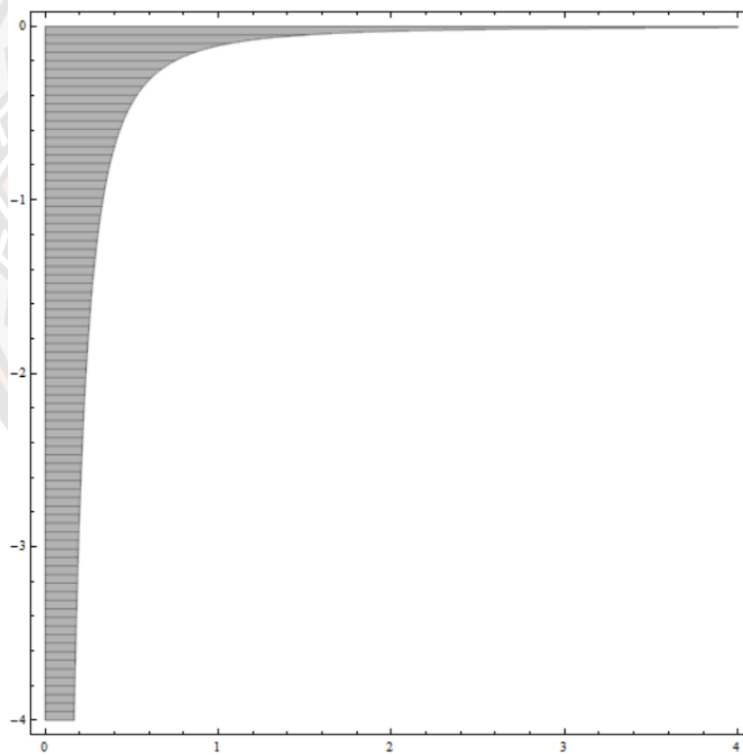


Figure 13 Existence diagram in wormholes with a cosmological constant by Λ and μ .

We choose $\Lambda = -0.1$ and $\mu = 0.5$

Volume of exotic matter is 59.651

In the same method ($P = 0, \Lambda = -0.1$ and $\mu = 0.5$), existence of wormhole in massive gravity show in diagram.

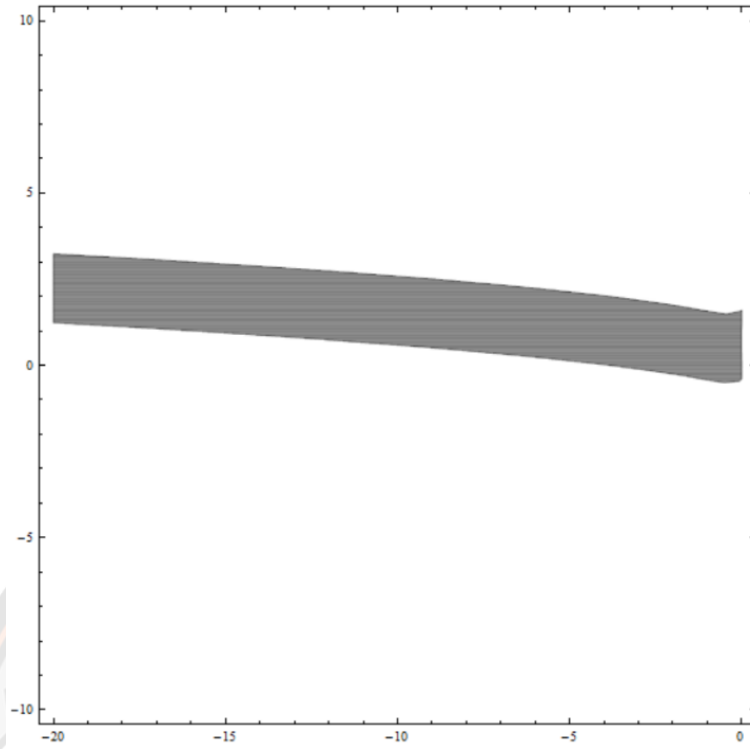


Figure 14 Existence diagram in wormholes in massive gravity by ζ and γ .

We can choose $\zeta = 1$ and $\gamma = -1$

Volume of exotic matter is 3.79193

Second shape function

$$b(r) = r_o^2/r$$

Let $P = 0$ (maximum surface radius), surface radius of wormholes with a cosmological constant can be obtained

$$a_c = \left(-\frac{3\mu}{\Lambda} \right)^{1/3}$$

The throat of wormholes with a cosmological constant can be obtained

$$r_{oc} = \left(-\frac{(-3\mu)^{4/3}}{\Lambda^{1/3}} \right)^{1/2}$$

For existence of wormholes ($a_c \geq r_{oc}$)

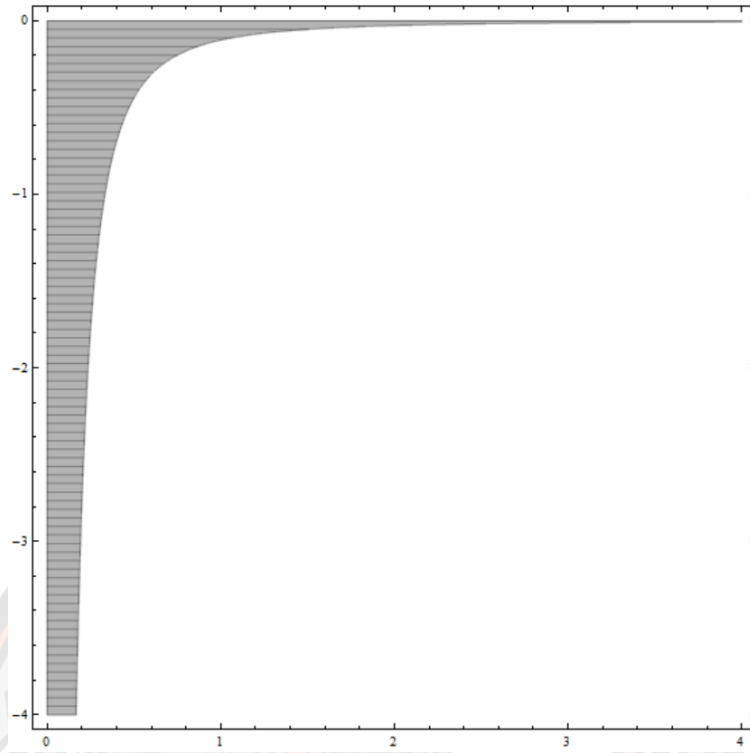


Figure 15 Existence diagram in wormholes with a cosmological constant by Λ and μ .

We choose $\Lambda = -0.1$ and $\mu = 0.5$

Volume of exotic matter is 33.0281

In the same method ($P = 0, \Lambda = -0.1$ and $\mu = 0.5$), existence of wormhole in massive gravity show in diagram.

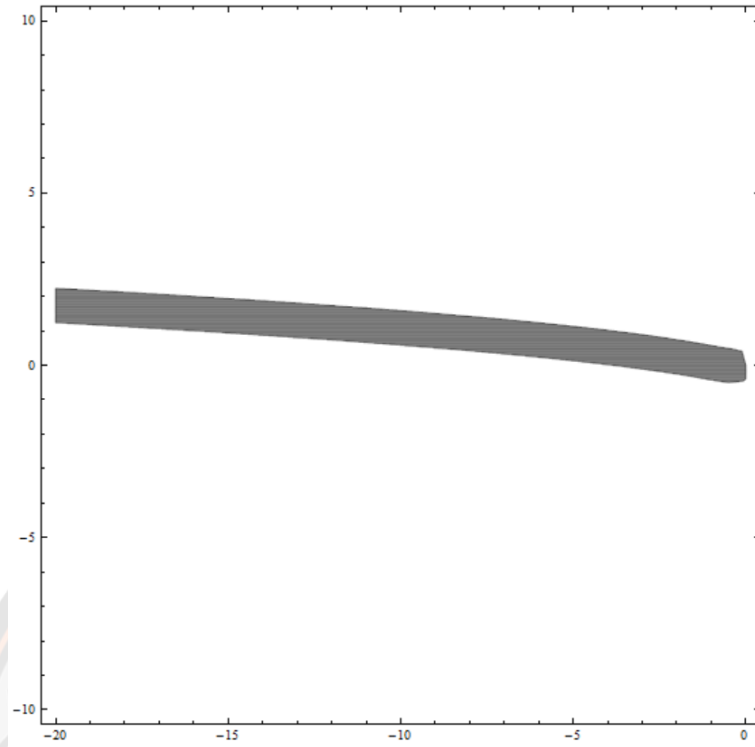


Figure 16 Existence diagram in wormholes in massive gravity by ζ and γ .

We can choose $\zeta = 1$ and $\gamma = -1$

Volume of exotic matter is 3.81331

For $b(r) = (r_0 r)^{1/2}$, the volumes of exotic matter in GR with cosmological constant and dRGT model are 59.65 and 3.79 respectively. For $b(r) = r_0/r$, volumes in GR with cosmological constant and dRGT model are 33.03 and 3.81 respectively. These mean that, in order to construct TWH, the usage of exotic matter in dRGT model can be less than one in GR with cosmological constant.

CHAPTER V

CONCLUSION

Even though TWH can solve the traversable wormhole's problem, there is still an issue about the negative radial pressure of the exotic matter which is called tension. Because of this issue, the traversable wormhole's problem is not completely solved yet. Hence, in our research, this problem is reduced by reducing the usage of the exotic matter. We expect that there might exist the NEC violation in our universe. Another required condition is that TWH should not be large because it have not been observed yet at the present.

The properties of exotic matter in TWH in dRGT model are found in the section 3. Three main regions of spacetime are the exterior (vacuum), interior (filled by exotic matter with negative radial pressure) regions and thin shell. The redshift function in interior region is set as $\Phi' = 0$ (zero gravity), which implies the weakness of gravitation field and tidal force. It is found that the thin shell is not needed in order to construct TWH in dRGT model (differ to GR case). By the condition of absence of thin shell, one can find the suitable ranges of parameters.

The results show that the comparisons for the volumes of exotic matter between TWH in GR with cosmological constant and dRGT model are checked. TWH in dRGT model can use less exotic matter than one in GR with cosmological constant. The figure showed that it is possible to construct a small-size TWH with tension approaching to zero. One can conclude that TWH in dRGT model is better with two mentioned reasons.

This research shows that the Massive Gravity theory yields TWH with required properties, and, if this theory can explain some observed phenomena, TWH might be observed in our universe.

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APPENDIX

Existence of wormholes

Set $P = 0$ (without thin shell because we find the largest surface radius) and find $a > r_o$ (surface radius must be larger than the throat)

dS Existence

We set $\mu = 1$ and $\Lambda = -0.1$ and calculate to plot a region diagram.

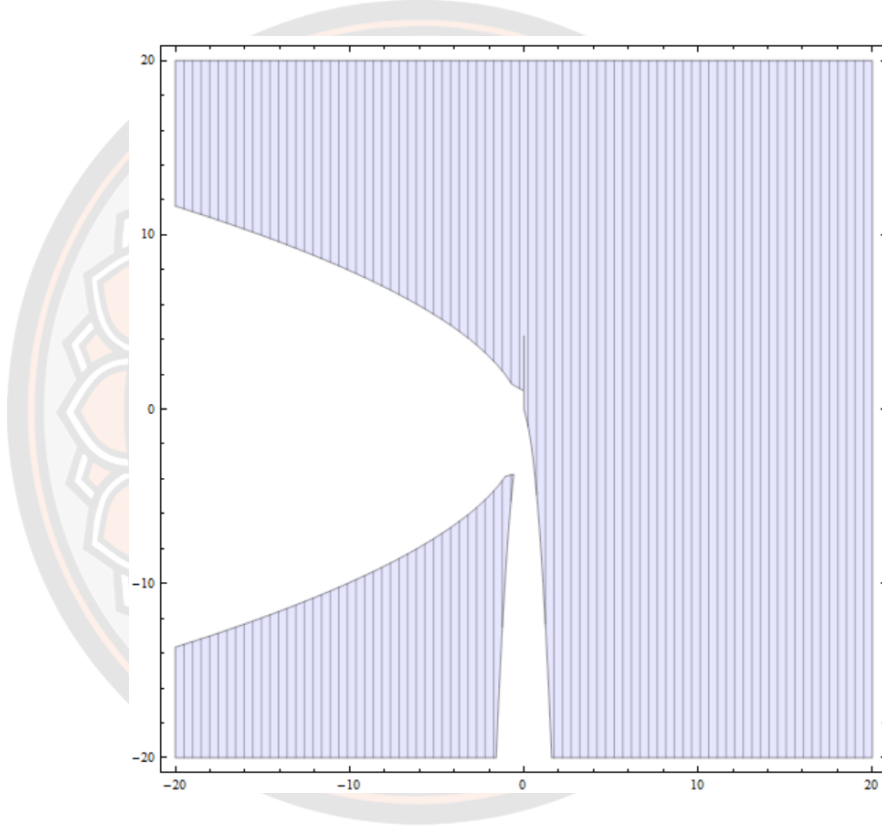


Figure 17 dS existence diagram by ζ and γ .

AdS Existence

We set $\mu = 1$ and $\Lambda = 0.1$ and calculate to plot a region diagram.

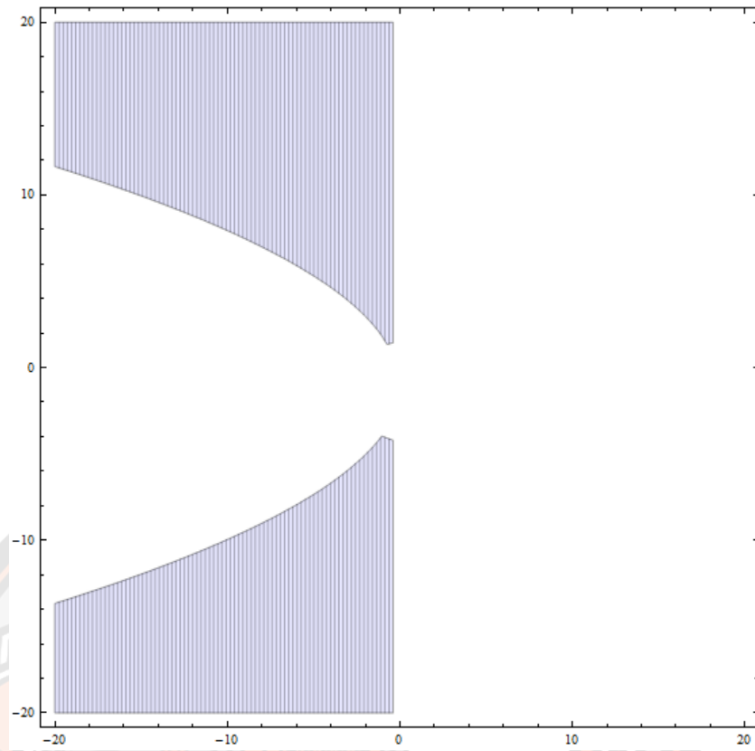


Figure 18 AdS existence diagram by ζ and γ .

Horizons

From exterior solution (empty spacetime), horizons can be obtained by solving a equation $g^{11} = 0$.

2 horizons

We set $\mu = 1$ and $\Lambda = -0.1$ (for dS)

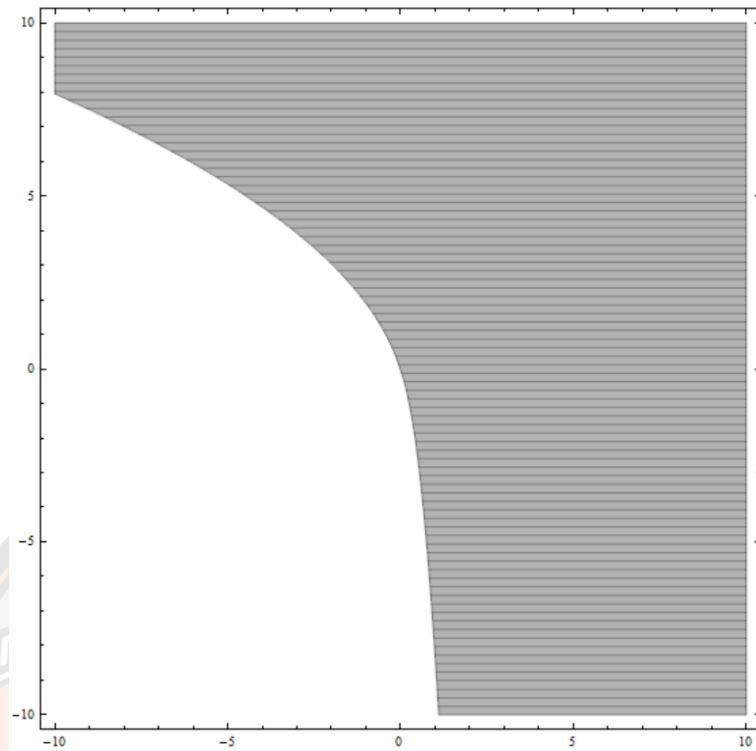


Figure 19 A diagram show region which is the solution with 2 horizons by ζ and γ .

3 horizons

We set $\mu = 1$ and $\Lambda = 0.1$ (for AdS)

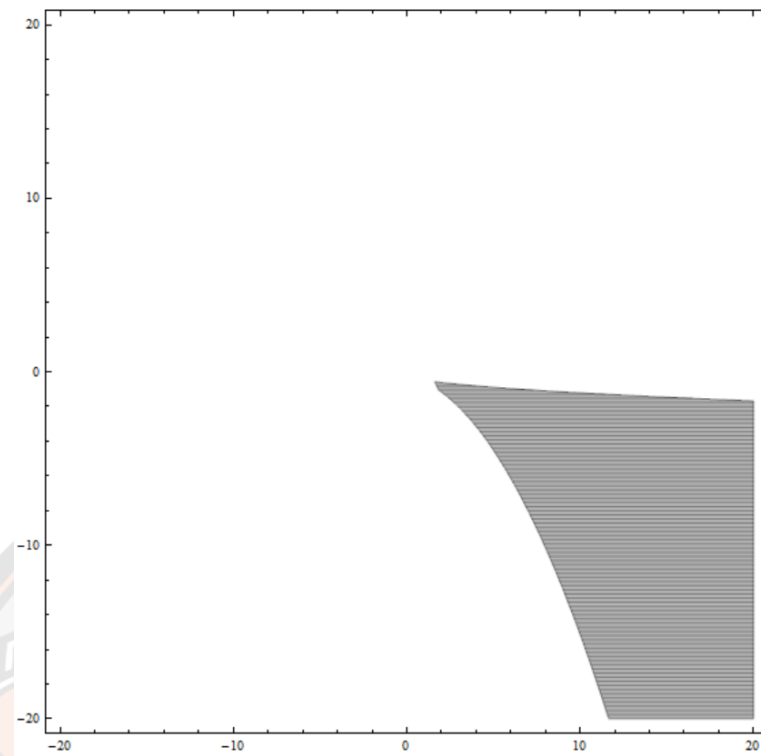


Figure 20 A diagram show region which is the solution with 3 horizons by ζ and γ .