สัญญาเลขที่ R2554B072

รายงานการวิจัยฉบับสมบูรณ์

โครงการวิจัยทฤษฎีหายนะพิบัติ
แผนภาพไฟน์แมนน์กับการประยุกต์ใช้ในฟิสิกส์และระบบสัญญาณ

ผู้วิจัย

นายบุรินทร์ กำจัดภัย

วิทยาลัยเพื่อการค้นคว้าระดับรากฐาน
“สถาบันสำนักเรียนท่าโพธิ์ฯ”
มหาวิทยาลัยนเรศวร

สนับสนุนโดยสำนักงานคณะกรรมการการวิจัยแห่งชาติ
และมหาวิทยาลัยนเรศวร

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(ความเห็นในรายงานนี้เป็นของผู้วิจัย วช. และมหาวิทยาลัยนเรศวรไม่จำเป็นต้องเห็นด้วยเสมอไป)

16 พฤษภาคม พ.ศ. 2556

กิตติกรรมประกาศ

ผู้วิจัยขอขอบคุณสำนักงานคณะกรรมการการวิจัยแห่งชาติและมหาวิทยาลัยนเรศวรสำหรับงบประมาณสนับสนุนโครงการวิจัย ขอขอบคุณ รศ. ดร. ปราโมทย์ วาดเขียน สำหรับคำแนะนำ

บทคัดย่อ

รหัสโครงการ: **R2554B072**

ชื่อโครงการ:

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ระยะเวลาโครงการ: 3 ปี (1 ธันวาคม 2553 ถึงวันที่ 30 พฤษภาคม 2556)

Linear oscillation systems, for example, forced mechanical vibration, series RLC circuit and parallel RLC circuit can be solved by using simplest initial conditions or employing of Green's function method of which knowledge of initial condition of the particular solution is needed. Here we propose an alternative method from quantum mechanics to solve this problem. By using Born approximation to the particular solution at initial state, we obtain infinite Born series of the general solution. The solution can be expressed as a series of one-dimensional Feynman diagrams. We perform this perturbative analysis in light damping-forced oscillation case.

คำหลัก: Linear oscillation system, forced vibration, series RLC circuit and parallel RLC circuit, Born approximation, Feynman diagram

Abstract

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Project Title:

Catastrophe theory, Feynmann diagram and applications in physics and signal system

Principle investigator:

Dr. Burin Gumjudpai

TPTP

The Institute for Fundamental Study, Naresuan University

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Project Period: 3 years (1 December 2009 - 30 May 2013)

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Executive Summary

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- **การตีพิมพ์เผยแพร่**

ผลผลิตจากโครงการคาดว่าจะได้รับการตีพิมพ์ในวารสารวิชาการระดับนานาชาติจำนวน 1 บทความคือ

- Burin Gumjudpai (IF, Naresuan University). A perturbative analysis of forced-oscillation system

- การสร้างกลุ่มวิจัยและเครือข่ายวิจัย

การดำเนินกิจกรรมในโครงการนี้ส่งผลให้เกิดการทำงานร่วมกันกับนักวิจัยที่ภาควิชาวิศวกรรมโทรคมนาคม สถาบันเทคโนโลยีพระจอมเกล้าเจ้าคุณทหารลาดกระบัง

สารบัญ

บทที่ 1

Introduction

Dynamical oscillation appears in large class of natural phenomena. Typical examples are such as forced mechanical vibration in spring with damping and RLC circuits. These are modeled with linear ordinary differential equations. Linear oscillation systems, for example, forced mechanical vibration, series RLC circuit and parallel RLC circuit are solved to obtain solution using simplest initial conditions, i.e. $y(0) = 0$ and $\dot{y}(0) = 0$ where $\dot{x} \equiv \mathrm{d}x/\mathrm{d}t$ so that the problem is simplified and then obtain general solution as an addition of complementary and particular solutions. Viewing the differential operators of the equation as linear operators, principle of superposition can be used. The equations therefore are solvable with Green's function method (see, for example Barton [1]). Solving for particular solution using Green's function already includes knowledge of initial condition within the particular solution. In dynamical oscillations, initial state in RLC circuit can be controlled to some values so that one can solve for the particular solution however it is not always possible to know form of particular solution in which we need to determine the initial state.

In non-relativistic quantum mechanics, scattering problem described by time-independent Schrödinger equation can give an alternative view of solving this problem. The method in quantum mechanics is to do Born approximation and solve for particular solution perturbatively. With this approximation, we do not need to exactly know the form of particular solution before hand, but instead we can use the complementary solution. This work offers alternative way of looking at oscillation problems such as forced-mechanical vibration and RLC circuits with the method in quantum mechanics. We start with discussion of basics of linear operator. Then we discuss linear time-invariance system, its examples and Green's function of the system. Afterwards, we show how time-independent Schrödinger Equation can be written in form of Helmholtz equation and its solution obtained from Green's function method. We later show how to apply Born approximation in scattering problem in elementary quantum mechanics. We then apply the same procedure to dynamical oscillation problem.

บทที่ 2

Linear system

Theory of physical systems are widely modeled with linear system. In dynamics, equation of motion can be viewed as a linear operator $\hat{\mathcal{L}}$ acting on a function $y(t)$ as

$$\hat{\mathcal{L}}y(t) = \mathcal{F}(t)\,. \tag{2.1}$$

where t is an independent variable. This equation is indeed an equation of motion in various fields. Having linear property of the operator, the principle of superposition automatically comes along with the operator, i.e. for two independent solutions, $c_1y_1(t)$ and $c_2y_2(t)$

$$\hat{\mathcal{L}}\left[c_1y_1(t) + c_2y_2(t)\right] = \hat{\mathcal{L}}\left[c_1y_1(t)\right] + \hat{\mathcal{L}}\left[c_2y_2(t)\right] = c_1\mathcal{F}_1(t) + c_2\mathcal{F}_2(t)\,, \tag{2.2}$$

where c_1, c_2 are constant. We can extend this to N number of solutions which can be infinite,

$$\hat{\mathcal{L}}\left[\sum_{i=1}^{N} c_iy_i(t)\right] = \sum_{i=1}^{N} c_i\mathcal{F}_i(t)\,. \tag{2.3}$$

As in (2.1) we can see that any solution $y(t)$ and any inhomogeneous part $F(t)$ of the operator $\hat{\mathcal{L}}$ can hence be express as

$$y(t) = \sum_{i=1}^{N} c_i y_i(t), \qquad \mathcal{F}(t) = \sum_{i=1}^{N} c_i \mathcal{F}_i(t). \tag{2.4}$$

We will consider examples of linear system in next sections.

บทที่ 3

Dynamics: the linear time-invariance system

A linear system that is time-invariance so-called linear time-invariance system (LTI) has a form of

$$\hat{\mathcal{L}} = a_n \frac{\mathrm{d}^n}{\mathrm{d}t^n} + a_{n-1} \frac{\mathrm{d}^{n-1}}{\mathrm{d}t^{n-1}} + \ldots + a_1 \frac{\mathrm{d}}{\mathrm{d}t} + a_0 . \tag{3.1}$$

Here we consider second-order LTI system, in form of

$$\hat{\mathcal{L}}\left[y(t)\right] = \left[\frac{\mathrm{d}^2}{\mathrm{d}t^2} + \frac{a_1}{a_2}\frac{\mathrm{d}}{\mathrm{d}t} + \frac{a_0}{a_2}\right] y(t) = \frac{\mathcal{F}(t)}{a_2} . \tag{3.2}$$

We set in this system, $a_1/a_2 \equiv 2\beta$, $a_0/a_2 \equiv \omega_0^2$ and $F(t) \equiv \mathcal{F}(t)/a_2$. Examples of second-order LTI system are such as forced-damped harmonic mechanical vibration, series and parallel RLC circuits (See Kelly [2] and Cha and Molinder [3]). In case of forced-damped harmonic mechanical vibration, $y(t)$ is displacement $x(t)$, $2\beta = b/m$ and $\omega_0^2 = s/m$ where m is mass of the oscillator, b is a resistance which is in unit of kg·sec^{-1} and s is Hooke's spring constant. For a series RLC circuit, $y(t)$ would be quantity of either electrical charge $q(t)$ or electrical current $i(t)$. $2\beta = R/L$ and $\omega_0^2 = 1/(LC)$. For a parallel RLC circuit, $y(t)$ is voltage $v(t)$. $2\beta = 1/(RC)$ and $\omega_0^2 = 1/(LC)$. β is Neper frequency. L is inductance. C is capacitance and R is electrical resistance.

Dynamical Parameters	Mechanical Vibration	Series RLC circuit	Parallel RLC circuit
displacement	displacement, $y(t)$	charge, $q(t)$ or current, $i(t)$	voltage, $v(t)$
inertia	m	L	C
resistance	b	R	$1/R$
elasticity	k	$1/C$	$1/L$
ω_0^2	k/m	$1/(LC)$	$1/(LC)$
2β	b/m	R/L	$1/(RC)$

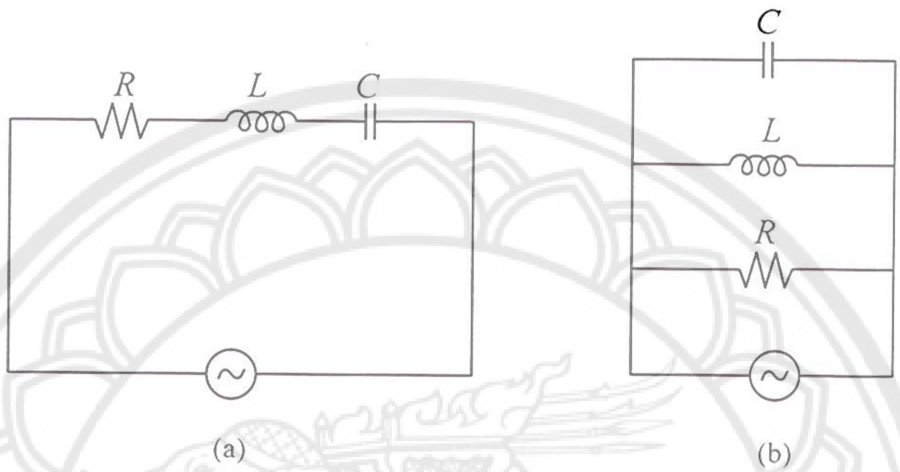
ตารางที่ 3.1: Dynamical parameters in second-order LTI systems for mechanical vibration, series RLC circuit and parallel RLC circuit.

Fig. 3.1 shows schematic diagram of these second-order LTI systems. Indeed we can conclude an analogy between three different linear systems: mechanical vibration, series and parallel RLC circuits as in Table 3.1. The equation (3.2) is hence

$$\frac{d^2y}{dt^2} + 2\beta\frac{dy}{dt} + \omega_0^2 y = F(t), \tag{3.3}$$

where the inertia (e.g. m) is absorbed into the function $F(t)$. This differential equation has general solution,

$$y(t) = y_c(t) + y_p(t). \tag{3.4}$$



รูปที่ 3.1: Schematic diagrams of (a) series RLC and (b) parallel RLC circuits

The complementary solution y_c is a solution of homogeneous system ($F(t) = 0$) and the particular solution is of inhomogeneous system, i.e. non-zero $F(t)$. As is well-known, when $F(t)$ is harmonic, i.e.

$$F(t) = F_0 \cos(\omega t - \phi) \tag{3.5}$$

the particular solution takes the form

$$y_p(t) = \frac{F_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2}} \cos(\omega t - \phi - \delta) , \tag{3.6}$$

with $\delta = \arctan\left[2\omega\beta/(\omega_0^2 - \omega^2)\right]$. Derivation is referred to standard textbooks, for instance, Main [4]. The $x_p(t)$ part represents steady-state solution which happens after some relaxation time τ_r. However, for other form of $F(t)$, in order to find $y_p(t)$, technique of Green's function is usually employed. This arises from the fact that there exists a Green's function satisfying this system with external force in form of Dirac's delta function.

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}t^2} + 2\beta\frac{\mathrm{d}}{\mathrm{d}t} + \omega_0^2\right] G(t) = \delta(t)\,. \tag{3.7}$$

This has solution,

$$y_{\mathrm{P}}(t) = \frac{b}{\omega_{\mathrm{d}}} e^{-\beta(t-t_0)} \sin\left(\omega_{\mathrm{d}}(t-t_0)\right), \quad t \geq t_0\,. \tag{3.8}$$

where $\omega_{\mathrm{d}} \equiv \sqrt{\omega_0^2 - \beta^2}$. The factor b can be replaced by R in the series RLC circuit system and $1/R$ of the parallel RLC circuit. The fact that response solution of linear oscillator to driving force in form of delta function can be found as equation (3.8), we can view arbitrary forcing function, i.e. the inhomogeneous part, as a series of impulses. This holds as long as the system is linear, i.e. the superposition principle is still valid. The solution hence can be expressed as

$$y_{\mathrm{P}}(t) = \int_{-\infty}^{t} G(t-t_0) F(t_0)\, \mathrm{d}t_0\,, \tag{3.9}$$

where Green's function for this system is

$$G(t-t_0) = \frac{1}{\omega_{\mathrm{d}}} e^{-\beta(t-t_0)} \sin\left(\omega_{\mathrm{d}}(t-t_0)\right) \quad \text{for} \quad t \geq t_0\,, \tag{3.10}$$

otherwise zero see results in standard classical dynamics textbooks e.g. Marion and Thornton [5] and Kibble and Berkshire [6]. The Green's function found here can be applied to any second-order LTI systems in form of equation (3.3). We stress that this can be applied to the system of both series and parallel RLC circuits as un mechanical vibration system.

บทที่ 4

Time-independent Schrödinger Equation: Helmholtz equation

In non-relativistic quantum mechanics, the equation of motion of particle for time-independent situation is the time-independent Schrödinger equation. The wave function ψ is only a function of spatial coordinate $\mathbf{r}$, that is $\psi = \psi(\mathbf{r})$. The Schrödinger equation reads

$$\frac{-\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r})\,. \tag{4.1}$$

This can be arranged to obtain

$$\left(\nabla^2 + k^2\right)\psi(\mathbf{r}) = \frac{2m}{\hbar^2}V(\mathbf{r})\psi(\mathbf{r}) \equiv Q(\mathbf{r})\,, \tag{4.2}$$

where here $k^2 \equiv 2mE/\hbar^2$ and m is mass of the particle. The Schrödinger equation is now expressed in form of the Helmholtz equation (4.2) (see, for example, in Barton

[1]). If there is a response solution. $G(\mathbf{r})$ to delta function $\delta^3(\mathbf{r})$.

$$\left(\nabla^2 + k^2\right) G(\mathbf{r}) = \delta^3(\mathbf{r}). \tag{4.3}$$

Then for an arbitrary inhomogeneous part-the "source" $Q(\mathbf{r})$, one can find particular solution $\psi_{\mathrm{p}}(t)$ as in Griffiths [7] and Schiff [8].

$$\psi_{\mathrm{p}}(\mathbf{r}) = \int_{-\infty}^{\mathbf{r}} G(\mathbf{r} - \mathbf{r}_0) Q(\mathbf{r}_0) \mathrm{d}^3\mathbf{r}_0 . \tag{4.4}$$

We can check the validity by substituting this solution $\psi_{\mathrm{p}}(\mathbf{r})$ into equation (4.2) to obtain,

$$\begin{aligned} \left(\nabla^2 + k^2\right) \psi_{\mathrm{p}}(\mathbf{r}) &= \int_{-\infty}^{\mathbf{r}} \left[\left(\nabla^2 + k^2\right) G(\mathbf{r} - \mathbf{r}_0)\right] Q(\mathbf{r}_0) \mathrm{d}^3\mathbf{r}_0 , \\ &= \int_{-\infty}^{\mathbf{r}} \delta^3(\mathbf{r} - \mathbf{r}_0) Q(\mathbf{r}_0) \mathrm{d}^3\mathbf{r}_0 , \\ &= Q(\mathbf{r}) . \end{aligned} \tag{4.5}$$

Similar procedure can be performed with equations (3.3), (3.7) and (3.9) to check validity of particular solution in the dynamical system in the last section. For the time-independent Schrödinger Equation in form of Helmholtz equation, Green's function has been known as,

$$G(\mathbf{r}) = -\frac{e^{ikr}}{4\pi r}. \tag{4.6}$$

When inhomogenous part is not presented, i.e. $V(\mathbf{r}) = 0$, the Green's function for this system is $G_0(\mathbf{r})$ and then

$$\left(\nabla^2 + k^2\right) G_0(\mathbf{r}) = 0 . \tag{4.7}$$

Adding equations (4.6) and (4.7) together, hence

$$\left(\nabla^2 + k^2\right) \left[G(\mathbf{r}) + G_0(\mathbf{r})\right] = \delta^3(\mathbf{r}). \tag{4.8}$$

Therefore one can always find complementary solution, $\psi_c(\mathbf{r})$ and hence general solution,

$$\psi(\mathbf{r}) = \psi_c(\mathbf{r}) + \psi_p(\mathbf{r}), \tag{4.9}$$

of the system. Using the equations (4.2), (4.4) and (4.6), the general solution (4.9) is therefore

$$\psi(\mathbf{r}) = \psi_c(\mathbf{r}) - \frac{m}{2\pi\hbar^2}\int_{-\infty}^{\mathbf{r}} \frac{e^{ik|\mathbf{r}-\mathbf{r}_0|}}{|\mathbf{r}-\mathbf{r}_0|} V(\mathbf{r}_0)\psi(\mathbf{r}_0)\,\mathrm{d}^3\mathbf{r}_0\,. \tag{4.10}$$

where the second term on the right-hand side (including of the minus sign) is $\psi_p(\mathbf{r})$. This equation is the integral form of Schrödinger Equation. We can think of $\psi_c(\mathbf{r})$ as a plane-wave function, $\psi_0(\mathbf{r})$, of a particle incoming to a heavy point of scattering at $\mathbf{r} = \mathbf{r}_0$ with scattering potential $V(\mathbf{r}_0)$. After scattering, considering distance very far from the scattering point, the "response" wave function is $\psi_p(t)$.

บทที่ 5

Born Approximation in quantum mechanics

The solution $\psi_{\mathrm{p}}(t)$ can be analyzed perturbatively. One well-known method is to use Born approximation in quantum mechanics (see e.g. Griffiths [7] and Schiff [8]). For simplicity, we express

$$g(\mathrm{r}) \equiv -\frac{m}{2\pi\hbar^2}\frac{e^{ikr}}{r} \tag{5.1}$$

which is proportional to Green's function, $G(\mathrm{r})$, the equation (4.10) is hence

$$\psi(\mathrm{r}) = \psi_0(\mathrm{r}) + \int_{-\infty}^{\mathrm{r}} g(\mathrm{r}-\mathrm{r}_0)V(\mathrm{r}_0)\psi(\mathrm{r}_0)\,\mathrm{d}^3\mathrm{r}_0\,. \tag{5.2}$$

Born approximation considers that at r_0 the incoming plane wave is not much affected by the potential hence,

$$\psi(\mathbf{r}_0) \approx \psi_0(\mathbf{r}_0)\,. \tag{5.3}$$

Hence

$$\psi(\mathbf{r}) \approx \psi_0(\mathbf{r}) + \int_{-\infty}^{\mathbf{r}} g(\mathbf{r}-\mathbf{r}_0)V(\mathbf{r}_0)\psi_0(\mathbf{r}_0)\,\mathrm{d}^3\mathbf{r}_0\,. \tag{5.4}$$

It is sensible to write down $\psi_0(\mathbf{r}_0)$ as a scattered wave from $\mathbf{r}_{00}$ with the incoming wave $\psi_{00}(\mathbf{r}_0)$, hence

$$\psi_0(\mathbf{r}_0) = \psi_{00}(\mathbf{r}_0) + \int_{-\infty}^{\mathbf{r}_0} g(\mathbf{r}_0-\mathbf{r}_{00})V(\mathbf{r}_{00})\psi_0(\mathbf{r}_{00})\,\mathrm{d}^3\mathbf{r}_{00}\,. \tag{5.5}$$

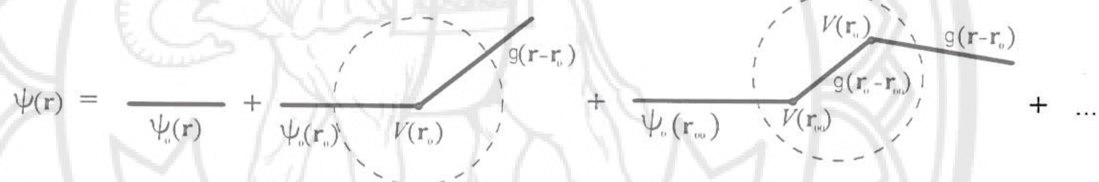
That is to say, the plane wave was scattered once at $\mathbf{r}_{00}$ by $V(\mathbf{r}_{00})$ before arriving at $\mathbf{r}_0$. Inserting equation (5.5) to (5.4) to obtain

$$\begin{aligned}\psi(\mathbf{r}) &= \psi_0(\mathbf{r}) + \int_{-\infty}^{\mathbf{r}} g(\mathbf{r}-\mathbf{r}_0)V(\mathbf{r}_0)\psi_{00}(\mathbf{r}_0)\,\mathrm{d}^3\mathbf{r}_0 \\ &\quad + \int_{-\infty}^{\mathbf{r}}\int_{-\infty}^{\mathbf{r}_0} \Big[g(\mathbf{r}-\mathbf{r}_0)V(\mathbf{r}_0)\Big]\Big[g(\mathbf{r}_0-\mathbf{r}_{00})V(\mathbf{r}_{00})\Big]\psi_0(\mathbf{r}_{00})\,\mathrm{d}^3\mathbf{r}_{00}\,\mathrm{d}^3\mathbf{r}_0\end{aligned} \tag{5.6}$$

The wave function in the second term on the right-hand side, $\psi_{00}(\mathbf{r}_0)$, with Born approximation, becomes $\psi_0(\mathbf{r}_0)$. With $\psi_{00}(\mathbf{r}_0) \approx \psi_0(\mathbf{r}_0)$, the result is

$$\begin{aligned}\psi(\mathbf{r}) &= \psi_0(\mathbf{r}) && (0^{\text{th}} \text{ order}) \\ &\quad + \int_{-\infty}^{\mathbf{r}} g(\mathbf{r}-\mathbf{r}_0)V(\mathbf{r}_0)\psi_0(\mathbf{r}_0)\,\mathrm{d}^3\mathbf{r}_0 && (1^{\text{st}} \text{ order}) \\ &\quad + \int_{-\infty}^{\mathbf{r}}\int_{-\infty}^{\mathbf{r}_0} \Big[g(\mathbf{r}-\mathbf{r}_0)V(\mathbf{r}_0)\Big]\Big[g(\mathbf{r}_0-\mathbf{r}_{00})V(\mathbf{r}_{00})\Big]\psi_0(\mathbf{r}_{00})\,\mathrm{d}^3\mathbf{r}_{00}\,\mathrm{d}^3\mathbf{r}_0\,. && (2^{\text{nd}} \text{ order})\end{aligned} \tag{5.7}$$

The first term is a plane wave, ψ_0 without scattering. In the second term, ψ_0 is scattered once at $\mathbf{r}_0$. The third term represents the incoming plane wave ψ_0 scattered twice, first at $\mathbf{r}_{00}$ and then at $\mathbf{r}_0$. We can extend this approximation beyond second order and the series becomes infinite series which is known as Born series. Detail discussion about the topics can be found in standard textbooks e.g. [7] and [8]. We can draw this infinite series in spirit of Feynman diagram in Fig. 5.1.



รูปที่ 5.1: Quantum mechanical-scattering Born series written as Feynman diagrams in two dimensions of space.

บทที่ 6

Born Approximation for a transient oscillation state

LTI system for a series RLC circuit can be transformed to the Helmholtz equation under transformation,

$$i(t) = I(t)\,e^{-tR/2L} = I(t)\,e^{-\beta t}. \tag{6.1}$$

This was reported recently by Sumichrast (2012) [9] where Fourier and Laplace transforms are applied to derive transient solution with designed initial conditions. Here we instead apply other technique in physics which is a perturbative method-the Born approximation to the problem. The approximation is standard and well-known in quantum mechanics and other scattering problems. It can be applied to the second-order LTI system. Let us start by considering driving-force term $F(t)$, which is an inhomogeneous part, as a product of a function $f(t)$ and a displacement $y(t)$.

$$F(t) \equiv f(t)y(t) . \tag{6.2}$$

Now we can view the function $f(t)$ in this setup as the potential term, $V(\mathbf{r})$ in quantum mechanics case. This is because $f(t)$ represents external influence on the system in similar manner to the way potential, $V(\mathbf{r})$ affect the scattering system. The solution $y(t)$ is general solution like the wave function $\psi(\mathbf{r})$. At time $t \geq t_0$, the vibration system is under influence of the driving-force term $F(t)$. The general solution (3.4) is now written as

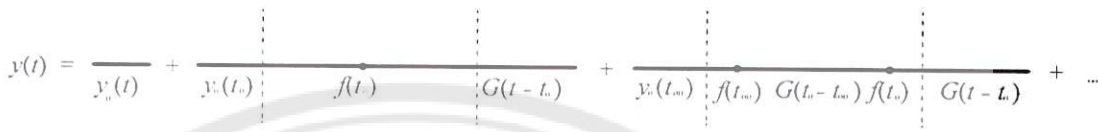
$$y(t) = y_c(t) + \int_{-\infty}^{t} G(t - t_0) f(t_0)\, y(t_0)\, \mathrm{d}t_0 \, . \tag{6.3}$$

We define $y_c(t) \equiv y_0(t)$ in analogous sense to quantum mechanical case. The Born approximation for vibration case is

$$y(t_0) \approx y_0(t_0) \tag{6.4}$$

This implies that at time $t = t_0$ the vibration is not much altered by effects of the external force. Using similar procedure to quantum mechanics case, we can find Born series for vibration system as,

$$\begin{aligned} y(t) &= y_0(t) && (0^{\text{th}} \text{ order}) \\ &+ \int_{-\infty}^{t} G(t - t_0) f(t_0) y_0(t_0)\, \mathrm{d}t_0 && (1^{\text{st}} \text{ order}) \\ &+ \int_{-\infty}^{t} \int_{-\infty}^{t_0} \Big[G(t - t_0) f(t_0)\Big] \Big[G(t_0 - t_{00}) f(t_{00})\Big] y_0(t_{00})\, \mathrm{d}t_{00}\, \mathrm{d}t_0 \, . && (2^{\text{nd}} \text{ order}) \\ &+ \text{ higher order terms } + \ \ldots . \end{aligned} \tag{6.5}$$



รูปที่ 6.1: Diagrams (in spirit of the Feynman diagram) of Born series for a signal (or vibrational) system in one dimension of time.

The Born series of the vibration system can not be drawn in more than one dimension. Notice that previously r is three-dimensional but now t has one dimension. The graphical representation is only one-dimensional straight line as in Fig. 6.1. We will consider application of this procedure in a problem of which the complementary and particular solutions are known.

บทที่ 7

Harmonic Driving Force: Light Damping Case Study

We choose harmonic driving force,

$$F = F_0 e^{i\omega t} . \tag{7.1}$$

The system considered here reads

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + 2\beta \frac{\mathrm{d}y}{\mathrm{d}t} + \omega_0^2 y = F(t) ,$$

which is equation (3.3). When it is homogenous ($F = 0$), the complementary solution is found to be,

$$y_0(t) = Ce^{pt} \tag{7.2}$$

where C is a constant and

$$p = -\beta \pm (\beta^2 - \omega_0^2)^{1/2} \tag{7.3}$$

The complementary solutions of the second-order LTI system appear in three cases. light damping. heavy damping and critical damping. These depend on the value of p. We do Born approximation

$$y(t_0) \approx y_0(t_0) \tag{7.4}$$

in our analysis just like in scattering case.

In light damping case. $\beta < \omega_0$ which yields $p = -\beta + i\,\omega_{\rm d}$. where we choose positive root. $\omega_{\rm d} = \sqrt{\omega_0^2 - \beta^2}$. The solution is

$$y_0(t) = Ae^{-\beta t + i\,\omega_{\rm d} t} \tag{7.5}$$

where $A = \exp(\phi_0)$ is a phase constant fixed by initial condition. We can write down the function $f(t)$ as

$$f(t) = \frac{F(t)}{y_0(t)} = \frac{F_0}{A} e^{\beta t}\, e^{i(\omega - \omega_{\rm d})t} \tag{7.6}$$

Using Born approximation,

$$y(t_0) \approx y_0(t_0) = Ae^{-\beta t + i\,\omega_{\rm d} t} \tag{7.7}$$

Hence the solution written as Born series is

$$\begin{aligned} y(t) &= y_0(t) + \int_{-\infty}^{t} \left(\frac{F_0}{\omega_{\rm d}}\right) e^{-\beta(t-t_0)} \Big\{ \sin[\omega_{\rm d}(t - t_0)]\, e^{i\omega t_0} \Big\}\, {\rm d}t_0 \\ &= + \int_{-\infty}^{t} \int_{-\infty}^{t_0} \frac{1}{A} \left(\frac{F_0}{\omega_{\rm d}}\right)^2 e^{-\beta(t-t_0-t_{00})} \Big\{ \sin[\omega_{\rm d}(t - t_0)]\, \sin[\omega_{\rm d}(t_0 - t_{00})]\, e^{i[(\omega-\omega_{\rm d})t_0 + \omega t_{00}]} \Big\}\, {\rm d}t_{00}\, {\rm d}t \end{aligned}$$

The part in curly bracket in the second term (first order in $F_0/\omega_{\rm d}$) manifests the transient state "beats" between two different frequencies, ω and $\omega_{\rm d}$. In the third term, part in curly bracket represents complex multiplying of two quantities:

$$\sin[\omega_{\rm d}(t - t_0)]\, e^{i(\omega-\omega_{\rm d})t_0} \qquad \text{and} \qquad \sin[\omega_{\rm d}(t_0 - t_{00})]\, e^{i\omega t_{00}}$$

The first one does not much look like beats due to its big difference in frequencies. However the second term could be the beats provided that ω_d does not differ much from the driving force frequency ω.

บทที่ 8

Conclusions

We have shown that Born approximation in quantum scattering can be used to find particular solution of dynamical oscillations which could be mechanical vibration, series RLC circuit and parallel RLC circuit. With the Born approximation, one does not need to know form of the particular solution but can approximate that, when force starts to exert on the system, the solution is not much altered from the complementary solution. This gives us some alternative analytic way of tackling the problem. Moreover, we hence can express the solution in Born series as graphical term in spirit of Feynman diagrams. We also show this analysis for a case of forced oscillation with light damping (underdamping).

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ภาคผนวก ก

ผลผลิตจากโครงการ

ก.1 การตีพิมพ์เผยแพร่ในวารสารวิชาการระดับนานาชาติ

ผลผลิตจากโครงการคาดว่าจะได้รับการตีพิมพ์ในวารสารวิชาการระดับนานาชาติจำนวน 1 บทความ คือ

- Burin Gumjudpai (IF, Naresuan University), A perturbative analysis of forced-oscillation system

ก.2 ผลลัพย์อื่นๆที่เกิดขึ้นจากการดำเนินโครงการ

ก.2.1 การสร้างกลุ่มวิจัยและเครือข่ายนักวิจัย

การดำเนินกิจกรรมในโครงการนี้ส่งผลให้เกิดการทำงานวิจัยร่วมกันในอนาคตกับนักวิจัยที่ภาควิชาวิศวกรรมโทรคมนาคม สถาบันเทคโนโลยีพระจอมเกล้าเจ้าคุณทหารลาดกระบัง